MATH MAMMOTH

Grade 6-A

Complete Worktext

- The basic operations and place value
- Ratios, proportions, and problem solving
- Decimals
- Number theory
- Fractions

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www.MathMammoth.com

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Foreword

Math Mammoth Grade 6-A and *Grade 6-B* worktexts comprise a complete math curriculum for the sixth grade mathematics studies. It contains eight main chapters. Each chapter has an introduction, which contains notes to the teacher; the actual lessons then follow with problems. Each chapter ends in a review lesson. The chapter tests are found in a separate folder and are printed separately. This product also includes an HTML page that you can use to make extra practice worksheets for computation.

Sixth grade is a time to deepen the student's understanding of fractions and decimals, and to study ratios, proportions, and percent as new topics. Also studied are integers, geometry, statistical graphs, and probability.

The *Math Mammoth Grade 6 Complete Curriculum* starts out with a review of the four operations with whole numbers and place value. Students are also introduced to exponents. While most of the first chapter is review, we also emphasize the ability to deal with expressions such as $600 \div 4 + 10^3$.

Next is ratios and proportions, which are a major part of study in sixth grade. Students are already somewhat familiar with ratios and finding fractional parts from earlier grades, and now it is time to advance that knowledge into the study of proportions, which arise naturally from the study of fractions and equivalent ratios. We show several ways of solving proportions. As applications, students study scaling geometric figures, floor plans, and scaling in maps.

The next several chapters deal with fractions and decimals. They have already been studied extensively in fifth grade. Now in sixth grade we review those topics, using more decimal digits (decimals) and larger denominators (fractions). Students also study factoring, the least common multiple, and the greatest common factor, and can use those concepts when dealing with fraction simplification or addition.

Percent is an important topic to understand thoroughly so we devote a whole chapter to it. It ties in with decimals, fractions, ratios, *and* proportions, and of course has lots of practical applications.

The study of integers is preparing students for pre-algebra and algebra. Here we study all four operations with integers, and also graphing simple functions.

In geometry, students encounter angle problems and calculations. We also study congruent transformations and similar figures. Area and volume calculations are reviewed from fifth grade. Pi and the area of a circle are studied as new topics.

Then, we turn our attention to statistics and probability. Students will analyze and create a variety of statistical graphs, and study mean, median, and mode. Probability is a topic that until recently was not a part of grade-school curriculum. Thus, we start slowly, introducing the probability topics in a simple manner. Students will encounter these topics again in pre-algebra and algebra.

I wish you success in your math teaching!

Maria Miller, the author

Chapter 1: Basic Operations and Place Value
Introduction

The goal of this first chapter is to review the four basic operations with whole numbers, review the order of operations and place value, and to learn about exponents, including the order of operations with exponents.

A lot of this chapter is just review, with the exception of the topic of exponents. This should hopefully provide a gentle start for 6th grade math where students are allowed to review some important topics, and yet learn something important as well.

Some major goals for 6th grade math that will be studied later are a mastery of all fraction and decimal operations, a solid understanding of ratio, proportion, and percent, and being able to solve common problems that involve these concepts.

The Lessons in Chapter 1

Helpful Resources on the Internet

Calculator Chaos
Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.
http://www.mathplayground.com/calculator_chaos.html

ArithmeTiles
Use the four operations and numbers on neighboring tiles to make target numbers.
http://www.primarygames.com/math/arithmetiles/index.htm

Choose Math Operation
Choose the mathematical operation(s) so that the number sentence is true. Practice the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.
http://www.homeschoolmath.net/operation-game.php

MathCar Racing
Keep ahead of the computer car by thinking logically, and practice any of the four operations at the same time.
http://www.funbrain.com/osa/index.html

SpeedMath Deluxe
Create an equation from the four given digits using addition, subtraction, multiplication and division. Make certain that you remember the order of operations.
http://education.jlab.org/smdeluxe/index.html

Fill and Pour
Fill and pour liquid with two containers until you get the target amount. A logical thinking puzzle.
http://nlvm.usu.edu/en/nav/frames_asid_273_g_2_t_4.html

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Powers and Exponents

Exponents are a kind of "shorthand" for writing repeated multiplications by the same number.

For example, $2 \times 2 \times 2 \times 2 \times 2$ is written 2^5.
$5 \times 5 \times 5 \times 5 \times 5 \times 5$ is written 5^6.

The tiny raised number is called the *exponent*. It tells us how many times the *base* number is multiplied by itself.

exponent

$$12^4 = 12 \times 12 \times 12 \times 12$$
$$= 20{,}736$$

base

The expression 2^5 is read "two raised to the fifth power," "two to the fifth power," or even just "two to the fifth." Similarly, 7^9 is read "seven raised to the ninth power," "seven to the ninth power," or "seven to the ninth." The "powers of 6" are simply expressions where 6 is raised to some power: For example, 6^3, 6^4, 6^{45}, and 6^{99} are powers of 6.

However, expressions with powers of 2 and 3 are almost always read differently:
The expression 11^2 is usually read as "eleven squared" because it describes the area of a square with sides 11 units long. Similarly, 31^3 is generally read as "thirty-one cubed" because it gives the volume of a cube with sides 31 units long.

1. Write out these expressions as multiplications, then solve them.

a. $3^2 = \underline{3 \times 3 = 9}$

b. 1^6

c. 4^3

d. 10^6

e. 5^3

f. 10^2

g. 2^3

h. 8^2

i. 0^3

j. 10^5

2. Rewrite these expressions as multiplication. Then use a calculator to solve them.

a. 6^4

b. 11^3

c. 13^3

d. 27^5

3. Rewrite each expression using an exponent, then solve it. You may use a calculator.

a. $2 \times 2 \times 2 \times 2 \times 2 \times 2$

b. $8 \times 8 \times 8 \times 8 \times 8$

c. 40 squared

d. $10 \times 10 \times 10 \times 10$

e. nine to the eighth power

f. eleven cubed

The expression 7^2 is read "seven *squared*" because it tells us the area of a *square* with sides 7 units long.

For example, if the sides of a square are 5 cm long, then its area is 5 cm × 5 cm = 25 cm^2.

Notice that the symbol for "square centimeters" is **cm^2**. This means "centimeter × centimeter." We are, in effect, squaring the measuring unit! In fact, we do the same thing when we use the units "square meters" and "square kilometers."

We could also write that expression as $(5\ cm)^2$ or "the quantity, five centimeters, squared." This means that both the 5 and the unit "cm" are squared, which makes 25 cm^2. Without the parenthesis it would be 5 cm^2 and mean "five square centimeters," which is something very different.

We can do the same thing with the traditional units of inches, feet, and miles. People often write "sq. in." for square inches, or "sq. ft." for square feet, instead of **in^2** and **ft^2**, but both ways are correct.

Similarly, 7^3 is read "seven *cubed*" because it gives the volume of a *cube* with sides 7 units long.

For example, if the sides of a cube are 10 cm long, then its volume is $(10\ cm)^3$ = 1,000 cm^3, or "one thousand cubic centimeters.".

4. Express the area using exponents and solve.

a. A square with a side of 12 kilometers: The area is $(12\ km)^2$ = _____	**b.** A square with sides 6 m long: Its area is
c. A square with sides each 6 inches long: Its area is	**d.** A square with a side with a length of 12 ft: The area is

5. Express the volume using exponents and solve.

a. A cube with a side of 2 cm: The volume is	**b.** A cube with sides each 10 inches long:
c. A cube with sides 1 ft in length:	**d.** A cube with edges that are all 5 m long:

6. **a.** The perimeter of a square is 40 cm. What is its area?

b. The volume of a cube is 64 cubic inches. How long is its side?

c. The area of a square is 121 m^2. What is its perimeter?

d. The *area* of one face of a cube is 64 in^2. What is its volume?

Notice something special about powers of 0 and powers of 1.

$0^5 = 0 \times 0 \times 0 \times 0 \times 0$ is simply 0! You can easily see that 0^3 , 0^7 , 0^{21}, and all of the other powers of 0 (0 raised to any whole-number power) are equal to 0.

$1^6 = 1 \times 1 \times 1 \times 1 \times 1 \times 1$ is simply 1! It's easy to see that 1^3 , 1^9 , 1^{65}, and all of the other powers of 1 (1 raised to any whole-number power) are equal to 1.

The powers of 10 are also very special—and very easy!

Notice that the exponent tells us *how many zeroes* there are in the answer.

$10^1 = 10$	$10^4 = 10{,}000$
$10^2 = 10 \times 10 = 100$	$10^5 = 100{,}000$
$10^3 = 10 \times 10 \times 10 = 1{,}000$	$10^6 = 1{,}000{,}000$

7. Fill in the patterns. In part (d), choose your own number to be the base.
Use a calculator in parts (c) and (d).

a.	**b.**	**c.**	**d.**
$2^1 =$	$3^1 =$	$5^1 =$	
$2^2 =$	$3^2 =$	$5^2 =$	
$2^3 =$	$3^3 =$	$5^3 =$	
$2^4 =$	$3^4 =$	$5^4 =$	
$2^5 =$	$3^5 =$	$5^5 =$	
$2^6 =$	$3^6 =$	$5^6 =$	

8. Look at the patterns above. Think carefully about how each step comes from the previous one.
Then answer the questions.

a. If you are given that $3^7 = 2{,}187$,
how can you use that result to find 3^8 ?

b. Find 3^8 without a calculator.

c. If you are given that $2^{45} = 35{,}184{,}372{,}088{,}832$,
how can you use that result to find 2^{46} ?

d. Find 2^{46} without a calculator.

Puzzle Corner

Make a pattern, called a *sequence*, with the powers of 2, starting with 2^6 and going *backwards* to 2^0. At each step, *divide* by 2. What is the logical (though surprising) value for 2^0 from this method? Make another, similar, sequence for the powers of 10. Start with 10^6 and divide by 10 until you reach 10^0. What value do you calculate for 10^0?
Try this same pattern for at least one other base number, n. What value do you calculate for n^0?
Do you think it will come out this way for every base number? Why or why not?

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Chapter 2: Ratios and Proportions
Introduction

In this chapter we concentrate on two important concepts: ratios and proportions and word problems involving those. We also study a little bit about expressions, equations, and graphing simple functions—elementary algebra concepts—but only on an introductory level. All of these topics are important in further studies of algebra.

This chapter has quite a bit of reading to do, probably more than in some other math curricula. This is because the text will often explain *why* the mathematical process works. Another reason is because we do a lot of problem solving in this chapter.

We start out by studying expressions and very simple equations. This is just a brief introduction to algebra, and these two topics are definitely studied a lot more in prealgebra and algebra courses. Now, ratios and proportions are *also* studied in prealgebra and algebra courses, so they will be reviewed in subsequent grades. However, my aim is to provide students with a thorough understanding of ratios and proportions here in 6th grade, not only because that is the norm, but also because they are used so much in everyday-life applications, and because they are a natural extension to go to after the student understands the basics of fractions.

After the little bit of expressions and equations, we study ratios, which should be a familiar topic from *Math Mammoth 5th grade*. The focus is on equivalent ratios because those will lead us into proportions just a few lessons later. After the introduction to ratios, we study various kinds of word problems involving ratios, and use a block or bar model to solve these problems. The lesson *Ratios in Rectangles* has applications about the aspect ratio.

Solving proportions is divided into three separate lessons. In the first one, we solve proportions by thinking through equivalent ratios. In the second one, the usual method of cross-multiplying is introduced. Then follows a lesson that explains just why cross-multiplying is allowed. Lastly, there is more practice with solving proportions and word problems.

As a last topic in the chapter, we study scaling geometric figures and floor plans, which are simple applications of proportions.

The Lessons in Chapter 3

Helpful Resources on the Internet

Equation Match
Playing on level 1, you need to match simple equations based on them having the same solution.
http://www.bbc.co.uk/education/mathsfile/shockwave/games/equationmatch.html

Algebraic Reasoning
Find the value of an object based on two scales.
http://www.mathplayground.com/algebraic_reasoning.html

Algebra Puzzle
Find the value of each of the three objects presented in the puzzle. The numbers given represent the sum of the objects in each row or column.
http://www.mathplayground.com/Algebra_Puzzle.html

Battleship
Choose the right solution for a 1-step equation every time you hit the enemy's ship. Some of the equations involve negative solutions; however since the game is interesting, some students might be willing to play it anyway (you can always guess at the right solution since it is a multiple choice game).
http://www.quia.com/ba/36544.html

Algebra Meltdown
Solve simple equations using function machines to guide atoms through the reactor. But don't keep the scientists waiting too long or they blow their tops. Again, includes negative numbers.
http://www.mangahigh.com/en_gb/games/algebrameltdown

Words into Equations Battleship Game
Practice expressions such as quotient, difference, product, and sum.
http://www.quia.com/ba/210997.html

Balance when Adding and Subtracting Game
The interactive balance illustrates simple equations. Your task is to add or subtract x's, and add or subtract 1's until you have x alone on one side.
http://www.mathsisfun.com/algebra/add-subtract-balance.html

Algebra Balance Scales
Similar to the one above, but you need to first put the x's and 1's in the balance to match the given equation.
http://nlvm.usu.edu/en/nav/frames_asid_201_g_4_t_2.html — only positive numbers
http://nlvm.usu.edu/en/nav/frames_asid_324_g_4_t_2.html — includes negative numbers

Practice with Ratios
An online quiz from Regents Exam Prep Center
http://www.regentsprep.org/Regents/math/ALGEBRA/AO3/pracRatio.htm

Practice with Proportions
An online quiz from Regents Exam Prep Center
http://www.regentsprep.org/Regents/math/ALGEBRA/AO3/pracProp.htm

Ratio Stadium
A multi-player online racing game for matching equivalent ratios. The student with the fastest rate of correct answers will win the race.
http://www.arcademicskillbuilders.com/games/ratio-stadium/

Dirt Bike Proportions
A racing game where you need to find the unknown in a simple proportion. This game would actually work equally well for practicing equivalent fractions, because the proportions are quite simple.
http://www.arcademicskillbuilders.com/games/dirt-bike-proportions/dirt-bike-proportions.html

Challenge Board
Choose questions from the challenge board about rates, ratios, and proportions.
http://www.quia.com/cb/158527.html

Ratio and Proportion Game From BBC Skillswise
Write the simplified ratio of red to black marbles. Answer simple questions about ratios and marbles.
http://www.bbc.co.uk/skillswise/numbers/wholenumbers/ratioandproportion/ratio/game.shtml

Ratio Pairs Matching Game
Match cards representing equivalent ratios.
Easy: http://nrich.maths.org/4824 Challenge: http://nrich.maths.org/4821

Equivalent Ratios Workout
10 online practice problems.
http://www.math.com/school/subject1/practice/S1U2L1/S1U2L1Pract.html

All About Ratios - Quizzes
Online quizzes about same and different ratios.
http://math.rice.edu/~lanius/proportions/index.html

Free Ride
An interactive activity about bicycle gear ratios. Choose the front and back gears, which determines the gear ratio. Then choose a route, pedal forward, and make sure you land exactly on the five flags.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=178

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Solving Problems Using Equivalent Ratios

Example. If Jake can ride his bike to a town that is 21 miles away in 45 minutes, how far can he ride in 1 hour?

Let's form an equivalent rate. However, it's not easy to go from 45 minutes to 60 minutes (1 hour). Therefore, let's first figure the rate for 15 minutes, which *is* easy.

Why? Because to get from 45 minutes to 15 minutes you simply divide both terms of the rate by 3.

$$\frac{21 \text{ miles}}{45 \text{ minutes}} \overset{\div 3}{\underset{\div 3}{=}} \frac{\square \text{ miles}}{15 \text{ minutes}} \overset{\times 4}{\underset{\times 4}{=}} \frac{\square \text{ miles}}{60 \text{ minutes}}$$

Then from 15 minutes, we can easily get to 60 minutes: Just multiply both terms by 4.

This is now easy to solve: $\frac{21 \text{ miles}}{45 \text{ minutes}} = \frac{7 \text{ miles}}{15 \text{ minutes}} = \frac{28 \text{ miles}}{60 \text{ minutes}}$. He can ride 28 miles in 1 hour.

1. If Jake can ride 8 miles in 14 minutes, how long will he take to ride 36 miles?
   Use the equivalent rates below.

$$\frac{8 \text{ miles}}{14 \text{ minutes}} = \frac{4 \text{ miles}}{\square \text{ minutes}} = \frac{36 \text{ miles}}{\square \text{ minutes}}$$

2. Write the equivalent rates.

a. $\frac{15 \text{ km}}{3 \text{ hr}} = \frac{\quad}{1 \text{ hr}} = \frac{\quad}{15 \text{ min}} = \frac{\quad}{45 \text{ min}}$

b. $\frac{\$6}{45 \text{ min}} = \frac{\quad}{15 \text{ min}} = \frac{\quad}{1 \text{ hr}} = \frac{\quad}{1 \text{ hr } 45 \text{ min}}$

c. $\frac{3 \text{ in}}{8 \text{ ft}} = \frac{\quad}{2 \text{ ft}} = \frac{\quad}{12 \text{ ft}} = \frac{\quad}{20 \text{ ft}}$

d. $\frac{115 \text{ words}}{2 \text{ min}} = \frac{\quad}{1 \text{ min}} = \frac{\quad}{3 \text{ min}}$

3. A car can go 50 miles on 2 gallons of gasoline.

   a. How many gallons of gasoline would the car need for a trip of 60 miles?
   Use the equivalent rates below.

$$\frac{50 \text{ miles}}{2 \text{ gallons}} = \frac{5 \text{ miles}}{\square \text{ gallons}} = \frac{60 \text{ miles}}{\square \text{ gallons}}$$

   b. How far can the car travel on 15 gallons of gasoline?
   (Hint: First consider the case with just 1 gallon of gasoline.)

Example. You get 20 erasers for $1.90.
How much would 22 erasers cost?

We cannot *easily* find a number by which to multiply to get from 20 to 22. It is possible, but we will solve this problem in a different way.

We will *first* figure out the cost of just 2 erasers! To find that, divide the price by 10. Then, from 2 erasers to 22 erasers, the cost will increase 11-fold:

11 × $0.19 = $2.09. So 22 erasers would cost $2.09.

Another way to come up with the same answer is just to add $0.19 (the cost of 2 erasers) to $1.90 (the cost of 20 erasers). Yet another way is to find the cost of 1 eraser first and multiply that cost by 22.

$$\frac{20 \text{ erasers}}{\$1.90} \overset{\times ??}{\underset{\times ??}{=}} \frac{22 \text{ erasers}}{?}$$

$$\frac{20 \text{ erasers}}{\$1.90} \overset{\div 10}{\underset{\div 10}{=}} \frac{2 \text{ erasers}}{\$\ 0.19} \overset{\times 11}{\underset{\times 11}{=}} \frac{22 \text{ erasers}}{\$\ \ }$$

4. On the average, Scott makes a basket nine times out of twelve shots when he is practicing. How many baskets can he expect to make when he tries 200 shots?

$$\frac{9 \text{ baskets}}{12 \text{ shots}} =$$

5. You get 30 pencils for $4.50.
How much would 52 pencils cost?

$$\frac{30 \text{ pencils}}{\$4.50} =$$

6. A train travels at a constant speed of 80 miles per hour.

a. How far will it go in 140 minutes? Use equivalent rates.

$$\frac{80 \text{ miles}}{1 \text{ hour}} =$$

b. How long will it take for the train to travel 50 miles? Use equivalent rates.

$$\frac{80 \text{ miles}}{1 \text{ hour}} =$$

7. In a poll of 1,000 people, 640 said they liked blue.

a. Simplify this ratio to the lowest terms.

b. Assuming the same ratio holds true in another group of 125 people, how many of those people can we expect to like blue?

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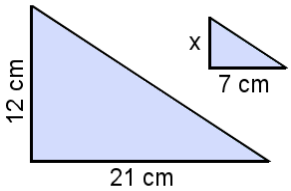
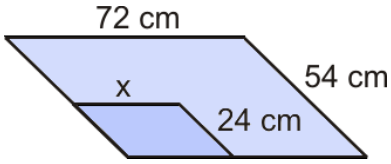
Scaling Figures 2

Example. The two triangles are similar. Find the side marked with *x*.

Here, we do *not* know the scale ratio, but we can solve it from the two corresponding sides that measure 7 cm and 21 cm. The scale is 21:7 or 3:1. Now we can figure out that the side length 12 cm needs to be divided by 3. We get *x* = 4 cm.



Example. The two parallelograms are similar. Find the length of the side marked by *x*.



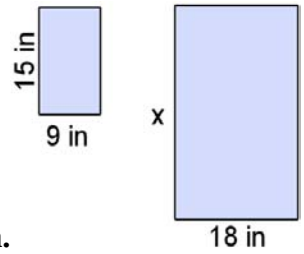
Solution 1. We can get the scale ratio from the two corresponding sides whose lengths are known—the 54-cm and 24-cm sides—and form a proportion to solve *x*. The ratio *x* : 72 has to equal the ratio 24 : 54.

$\frac{x}{72} = \frac{24}{54}$	We simplify the fraction 24/54 before continuing.
$\frac{x}{72} = \frac{4}{9}$	Now cross-multiply.
$9x = 4 \cdot 72$	Multiply the right side.
$9x = 288$	Divide both sides by 9.
$\frac{9x}{9} = \frac{288}{9}$	Divide the right side.
$x = 32$	So, x is 32 cm.

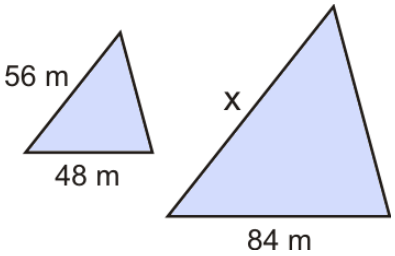
Solution 2. Like in solution 1, we first find and simplify the scale ratio using the two corresponding sides whose lengths are known. The scale ratio is 24:54 = 4:9 (simplified).

Now we use the scaling ratio 4:9 and compare it to the 72-cm side, which corresponds to the unknown side. Imagine that the 72-cm side is divided into 9 parts. Each part is 72 cm ÷ 9 = 8 cm. Similarly, the unknown side has four parts, so it is 4 × 8 cm = 32 cm.

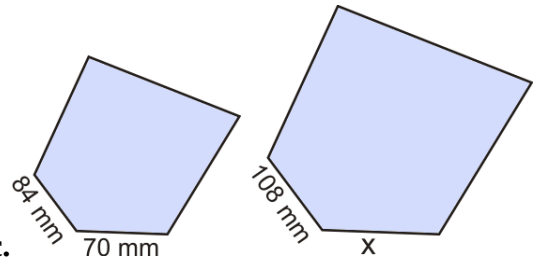
1. The figures are similar. Find the side lengths marked with *x*. First figure out the scale ratio.



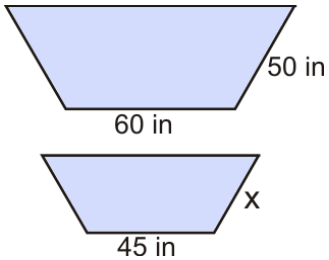
a.



b.

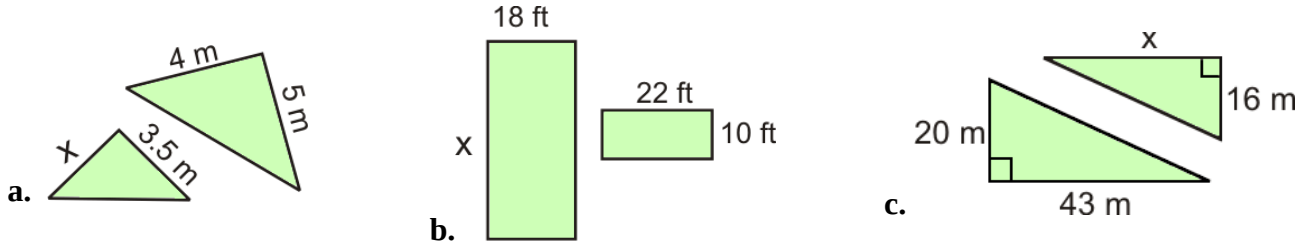


c.



d.

2. The figures are similar. Find the side lengths marked with *x*.



3. The sides of a rectangle measure 3″ and 4 1/2″.
The shorter side of another, similar rectangle is 3/4″.
How long is the longer side of that rectangle?

4. The rectangles 1 through 4 in the table on the right are similar.

	Length	Width
Rectangle 1	1 cm	
Rectangle 2	1.5 cm	
Rectangle 3	2 cm	
Rectangle 4	2.5 cm	7.5 cm

a. Follow the example given and find these ratios in whole numbers in lowest terms:

- The scale ratio between rectangle #3 and rectangle #4

  It is $\frac{2}{2.5} = \frac{4}{5}$.

- The scale ratio between rectangle #1 and rectangle #3

- The scale ratio between rectangle #1 and rectangle #4

b. Use the scale ratios to find the width of each rectangle, and fill in the rest of the table. Draw the rectangles in your notebook or on a separate sheet of paper.

c. What is the *aspect ratio* (*i.e.*, the ratio of length to width) of each rectangle?

5. The area of a square is 36 cm^2. The square is shrunk in a ratio of 4:3. What is the area of the resulting square?

6. A rectangle with the sides of 2 1/2 in. and 1 3/4 in. is enlarged in a ratio of 1:4. Find the area of the resulting rectangle.

7. The aspect ratio of a rectangle is 2:3 and its perimeter is 50 cm.
Find the area of the rectangle after it is shrunk in a scale ratio of 5:2.

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Chapter 3: Decimals
Introduction

In this chapter we study all four operations of decimals, the metric system, and using decimals in measuring units. Most of the topics here have already been studied in 5th grade, but the lessons and exercises were mostly using numbers with a maximum of three decimal digits. This time, there is no such restriction and the decimals used can have many more decimal digits than that. However, if the student has a good grasp of decimals already (for example because of having studied the *Math Mammoth Grade 5 Complete Curriculum* or through the *Math Mammoth Decimals 2* book), consider assigning only 1/3 - 1/2 of the problems because he should be able to go through this chapter quickly.

We start out by studying place value with decimals and comparing decimals, up to six decimal digits. The next several lessons contain mainly review, just using longer decimals than in 5th grade: adding and subtracting decimals, rounding decimals, using mental math for multiplying and dividing decimals, long division with decimals, fractions to decimals, and multiplying and dividing decimals by the powers of ten.

Scientific notation is a new topic. Most 7th grade math curricula also cover it. After that, we turn our attention one more time to dividing decimals by decimals. I have tried to explain the principle behind the common shortcut or rule ("Move the decimal point in both the divisor and the dividend so many steps that the divisor becomes a whole number"). The principle here has to do with multiplying the divisor and the dividend by a power of ten, and it even ties with equivalent fractions. Many school books never explain this principle in connection with decimal division.

The last lessons in this chapter deal with measuring units and the metric system, and nicely round up our study of decimals.

The Lessons in Chapter 3

Helpful Resources on the Internet

Place Value Strategy
Place the 3 or 4 digits given by the spinner to make the largest number possible.
www.decimalsquares.com/dsGames/games/placevalue.html

Decimal Darts
Try to pop balloons with darts by estimating the balloons' height.
www.decimalsquares.com/dsGames/games/darts.html

Decimal Challenge
Try to guess a decimal number between 0 and 10. Each time feedback tells you whether your guess was too high or too low.
www.interactivestuff.org/sums4fun/decchall.html

Beat the Clock
Type in the decimal number for the part of a square that is shaded in this timed game.
www.decimalsquares.com/dsGames/games/beatclock.html

Scales
Move the pointer to match the decimal number given to you. Refresh the page from your browser to get another problem to solve.
www.interactivestuff.org/sums4fun/scales.html

Switch
Put the sequence of decimal numbers into ascending order by switching them around. Refresh the page from your browser to get another problem to solve.
www.interactivestuff.org/sums4fun/switch.html

Smaller and Smaller Maze
Practice ordering decimal numbers to find your way through the maze.
www.mathsyear2000.org/magnet/kaleidoscope/smaller/index.html

Decimal and Whole Number Jeopardy
Review place value and comparing and rounding numbers. Also, practice number patterns.
www.quia.com/cb/8142.html

Decimals in Space
An Asteroids-style game where you first answer a question about the smallest decimal and then get to shoot asteroids, earning points based on the numbers on them.
themathgames.com/arithmetic-games/place-value/decimal-place-value-math-game.php

Sock
Push the green blocks into the holes to make the target number.
www.interactivestuff.org/sums4fun/sock.html

Decimal Squares Blackjack
Play cards with decimals, trying to get as close to 2 as possible without going over.
www.decimalsquares.com/dsGames/games/blackjack.html

A Decimal Puzzle
Make every circle add up to 3.
nlvm.usu.edu/en/nav/frames_asid_187_g_2_t_1.htmlsopen=instructions&from=category_g_2_t_1.html

FunBrain Decimal Power Football
Simple games for addition, subtraction, multiplication, and division of decimals, including some with a missing factor or divisor. Solve a problem, and the football player moves down the field.
funbrain.com

Exploring Division of Decimals
Use a square to explore the products of two numbers with one decimal digit. The product is shown as an area.
www.hbschool.com/activity/elab2004/gr6/1.html

Decimal Speedway
Practice decimal multiplication in this fun car-racing game.
www.decimalsquares.com/dsGames/games/speedway.html

Fractions - Decimals calculator
Convert fractions to decimals, or decimals to fractions, including repeating (recurring) decimals to any decimal places, which normal calculators don't do.
http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fractions/FractionsCalc.html

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Fractions to Decimals

One way to change a fraction into a decimal is to find an equivalent fraction with a denominator of 10, 100, 1000, etc. This works with *some* fractions, but not most.	$\frac{13}{20} \overset{\times 5}{\underset{\times 5}{=}} \frac{65}{100} = 0.65$	$\frac{27}{30} \overset{\div 3}{\underset{\div 3}{=}} \frac{9}{10} = 0.9$	$\frac{31}{125} \overset{\times 8}{\underset{\times 8}{=}} \frac{248}{1{,}000} = 0.248$

1. Write as decimals. Think of the equivalent fraction that has a denominator of 10, 100, or 1000.

a. $\frac{1}{4}$ **b.** $\frac{1}{5}$ **c.** $1\frac{1}{20}$ **d.** $3\frac{9}{25}$

e. $\frac{12}{200}$ **f.** $8\frac{3}{4}$ **g.** $4\frac{3}{5}$ **h.** $\frac{13}{20}$

i. $\frac{13}{25}$ **j.** $\frac{11}{125}$ **k.** $\frac{24}{400}$ **l.** $\frac{95}{500}$

2. Change to decimals and calculate mentally.

a. $0.2 + \frac{1}{4}$ **b.** $0.34 + 1\frac{1}{5}$ **c.** $2\frac{3}{5} + 1.3$ **d.** $\frac{2}{10} - 0.09$

When the method above doesn't work, simply remember that the fraction line indicates division and DIVIDE the numerator on top by the denominator on the bottom.

So to change, for example, $\frac{1}{7}$ or $\frac{25}{32}$ or $\frac{6}{17}$ into decimals, use long division (or a calculator).

Example. Find $\frac{18}{11}$ as a decimal. To do that, we write 18 as 18.0000 in the long division "corner" and divide by 11. We notice the digits "63" start repeating.

Therefore, $\frac{18}{11} = 1.636363...$

This can also be written by drawing a line over the repeating digits: $1.636363... = 1.\overline{63}$.

```
      0 1.6 3 6 3
11 )1 8.0 0 0 0
    1 1
      7 0
    - 6 6
        4 0
      - 3 3
          7 0
        - 6 6
            4 0
          - 3 3
              7
```

Example. Find $\frac{8}{31}$ to five decimal places. We cannot see any repeating pattern in the long division in the first six decimals. (There *is* a pattern, but it is 15 digits long!). Therefore, we stop the division after six decimals, and round the number to five decimals.

We get $\frac{8}{31} \approx 0.25806$.

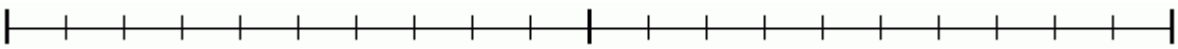
```
      0.2 5 8 0 6 4
31 )8.0 0 0 0 0 0
    6 2
    1 8 0
   -1 5 5
      2 5 0
    - 2 4 8
        2 0
      -   0
      2 0 0
    - 1 8 6
        1 4 0
        1 2 4
          1 6
```

3. Write as decimals. The ones you don't already know by heart, calculate by long division. Calculate to six places, then round to five, unless the division comes out even or you find a repeating pattern.

a. $\frac{2}{3}$	**b.** $1\frac{1}{3}$	**c.** $\frac{8}{9}$
d. $5\frac{17}{21}$	**e.** $\frac{19}{24}$	**f.** $\frac{1}{6}$
g. $\frac{3}{8}$	**h.** $2\frac{3}{11}$	**i.** $\frac{3}{7}$

4. Mark the following numbers on this number line that starts at 0 and ends at 2.

$$0.2,\ \frac{1}{4},\ 0.65,\ 1\frac{1}{3},\ 0.04,\ \frac{2}{5},\ 1.22,\ 1\frac{3}{4},\ 1.95,\ 1\frac{4}{5}$$



5. Write as decimals. Use a notebook to do long division. Not all of these are repeating decimals. Of those that are, can you find a pattern in the repeating parts of the decimals?

a.	**b.**	**c.**	**d.**
$1 \div 3 = 0.\overline{3}$	1 ÷ 9	1 ÷ 4	1 ÷ 6
2 ÷ 3	2 ÷ 9	2 ÷ 4	2 ÷ 6
3 ÷ 3	3 ÷ 9	3 ÷ 4	3 ÷ 6
4 ÷ 3	4 ÷ 9	4 ÷ 4	4 ÷ 6
5 ÷ 3	5 ÷ 9	5 ÷ 4	5 ÷ 6
6 ÷ 3	6 ÷ 9	6 ÷ 4	6 ÷ 6
7 ÷ 3	7 ÷ 9	7 ÷ 4	7 ÷ 6
e.	**f.**	**g.**	**h.**
$1 \div 7 = 0.\overline{142857}$	1 ÷ 8	1 ÷ 5	1 ÷ 11
2 ÷ 7	2 ÷ 8	2 ÷ 5	2 ÷ 11
3 ÷ 7	3 ÷ 8	3 ÷ 5	3 ÷ 11
4 ÷ 7	4 ÷ 8	4 ÷ 5	4 ÷ 11
5 ÷ 7	5 ÷ 8	5 ÷ 5	5 ÷ 11
6 ÷ 7	6 ÷ 8	6 ÷ 5	6 ÷ 11
7 ÷ 7	7 ÷ 8	7 ÷ 5	7 ÷ 11

6. Divide. All of these either have a repeating pattern, or the decimal terminates.

a. 34.4 ÷ 12

b. 66 ÷ 9

c. 0.76 ÷ 11

d. 0.23 ÷ 4

7. When the fraction $\frac{1}{3}$ is written as a decimal, it is 0.33333… .
This could be rounded to three decimals (0.333), or to six decimals (0.333333), or to any other amount of decimals.
Find the *difference* between the rounded versions, if 0.33333… is first rounded to five decimals and then to only two decimals.

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Metric System Prefixes

The metric system has one basic unit for each thing we might measure: For length, the unit is the **meter**. For weight, it is the **gram**. And for volume, it is the **liter**.

All of the other units for measuring length, weight, or volume are *derived* from the basic units using *prefixes*. These prefixes tell us what multiple of the basic unit the derived unit is.

Prefix	Abbreviated	Meaning
kilo-	k	1,000
hecto-	h	100
deca-	da	10
-	-	(the basic unit)
deci-	d	1/10
centi-	c	1/100
milli-	m	1/1000

Unit	Abbr	Meaning
kilometer	km	1,000 meters
hectometer	hm	100 meters
decameter	dam	10 meters
meter	m	(the basic unit)
decimeter	dm	1/10 meter
centimeter	cm	1/100 meter
millimeter	mm	1/1000 meter

Unit	Abbr	Meaning
kilogram	kg	1,000 grams
hectogram	hg	100 grams
decagram	dag	10 grams
gram	g	(the basic unit)
decigram	dg	1/10 gram
centigram	cg	1/100 gram
milligram	mg	1/1000 gram

Unit	Abbr	Meaning
kiloliter	kl	1,000 liters
hectoliter	hl	100 liters
decaliter	dal	10 liters
liter	l	(the basic unit)
deciliter	dl	1/10 liter
centiliter	cl	1/100 liter
milliliter	ml	1/1000 liter

1. Write these amounts in basic units (meters, grams, or liters) by "translating" their prefixes:
e.g., the "centi" in cm means "hundredths," so 3 cm is three hundredths of a meter (3/100 m).

a. 3 cm = 3/100 m = 0.03 m	**b.** 2 cg = _____ g = _____ g
5 mm = _____ m = _____ m	6 mL = _____ L = _____ L
7 dL = _____ L = _____ L	1 dg = _____ g = _____ g

2. Write the amounts in basic units (meters, grams, or liters) by "translating" the prefixes.

a. 3 kL = _____ L	**b.** 2 dam = _____ m	**c.** 70 km = _____ m
8 dag = _____ g	9 hL = _____ L	5 hg = _____ g
6 hm = _____ m	7 kg = _____ g	8 dal = _____ L

3. Write the amounts using derived units (prefixes).

a. 3,000 g = 3 kg	**b.** 0.01 m = _____ ____	**c.** 0.04 L = _____ ____
800 L = _____ ____	0.2 L = _____ ____	0.8 m = _____ ____
60 m = _____ ____	0.005 g = _____ ____	0.007 L = _____ ____

4. These measurements are all mixed up! Organize them in the table in descending order from the heaviest weight to the lightest.

Item	Mass

Grandpa 75 kg full suitcase
3 kg 30 kg
baby 200 g Cell phone
10 kg bucket of water

5. Write using prefixed units so that the number of units is the smallest possible whole number.

a. 0.04 meters = 4 cm

b. 0.005 grams

c. 0.037 meters

d. 400 liters

e. 0.6 meters

f. 2,000 meters

g. 0.206 liters

h. 20 meters

6. Change into the basic unit (either meter, liter, or gram). Think of the meaning of the prefix.

a. 45 cm = 0.45 m

b. 65 mg

c. 2 dm

d. 81 km

e. 6 mL

f. 758 mg

g. 2 kL

h. 8 dL

7. Find the total …

a. … weight of books that weigh individually:
1.2 kg, 1.04 kg, 520 g, and 128 g.

b. … volume of containers whose individual volumes are:
1.4 L, 2.25 L, 550 mL, 240 mL, and 4 dL.

8. Each measurement has a flub, either in the unit or in the decimal point. Correct them.

a. The length of a pencil: 13 m

b. The length of an eraser: 45 cm

c. Length of Dad's waist: 9.8 m

d. The height of a room: 0.24 m

e. Jack's height: 1.70 mm

f. Jenny's height: 1.34 cm

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Chapter 4: Number Theory
Introduction

Number theory has to do with the study of whole numbers and their special properties. In this chapter, we study divisibility rules, prime numbers, factoring, the greatest common factor (GCF), and the least common multiple (LCM).

The main application of factoring and the greatest common factor in arithmetic is in simplifying fractions, so that is why this chapter includes a lesson on that topic. However, it is not absolutely necessary to use the GCF when simplifying fractions, and the lesson emphasizes that fact. The really important, but far more advanced, application of prime numbers is in cryptography. Some students might be interested in reading additional material on that subject—please see the list below for Internet resources.

Similarly, the main use for the least common multiple in arithmetic is with the addition of fractions, and we study that topic in this chapter also.

The concepts of factoring and the GCF are also important to understand because they will be carried over to algebra, where students will factor polynomials.

Primes are fascinating "creatures", and you can let students read more about them using the Internet resources listed below.

The Lessons in Chapter 4

Helpful Resources on the Internet

Arrays and factors
Drag rectangles to show the factorizations of a given number.
http://www.shodor.org/interactivate/activities/factors2/index.html

Factor Game
A fun, interactive game where you practice divisibility among numbers 1-100. You can play against the computer or against a friend.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=12

Factors and Remainders
An interactive animation demonstrating factors and remainders. Choose a number and its possibe divisor. The animation shows boxes (as given by the number) arranged into rows of (possible divisor), and you can SEE if there is any remainder.
http://www.absorblearning.com/media/item.action?quick=ml

MathGoodies Interactive Factor Tree Game
Type in a missing number to the factor tree, and the program will find the other factor, and continue drawing the tree as needed.
http://www.mathgoodies.com/factors/prime_factors.html

Snake
Eat factors, multiples, and prime numbers in this remake of the classic game.
http://www.spacetime.us/arcade/play.php?game=2

Product game
For two players; each selects a factor, computer colors the product - who gets four in row wins.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=29

Primes, Factors and Divisibility - Explorer at CountOn.org
Lessons explaining divisibility tests, primes, and factors.
http://www.counton.org/explorer/primes/

Prime Number Calculator
This calculator tests if a number is a prime, and tells you its smallest divisor if it is not prime.
http://www.basic-mathematics.com/prime-number-calculator.html

The Prime Pages
Learn more about primes on this site: the largest known primes, finding primes, how many are there, and more.
http://primes.utm.edu/

The Cryptoclub. Using Mathematics to Make and Break Secret Codes (book)
Cryptoclub kids strive to break the codes of secret messages, and at the same time learn more and more about encrypting and decrypting. The book contains problems to solve at the end of each chapter, little tips, and historical information how cryptography has been used over the centuries. By solving the problems you can actually learn to do all of it yourself.
http://www.amazon.com/gp/product/156881223X?tag=homeschoolmat-20

Primality of 1 from Wikipedia
Discussing whether 1 should or should not be counted as a prime number.
http://en.wikipedia.org/wiki/Prime_number#Primality_of_one

Arguments for and against the primality of 1
http://primefan.tripod.com/Prime1ProCon.html

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Factoring and Primes 2

Some numbers have *only two* divisors: 1 and the number itself. Such numbers are called **prime numbers.** 11 is one of them.

factor		factor		product
1	×	**11**	=	**11**

In the last lesson, we found that the prime numbers between 1 and 20 are **2**, **3**, **5**, **7**, **11**, **13**, **17** and **19**. 1 is usually not counted as a prime number. (See the last lesson for an explanation of why not.)

Prime factorization using a factor tree

A factor tree is a handy way to factor numbers to their prime factors. The factor tree starts at the root and grows upside down!

We want to factor 24, so we write 24 at the top. Then we factor 24 into 4 × 6. But 4 and 6 are not prime numbers, so we *continue* to factor. We factor four into 2 × 2 and six into 2 × 3.

We can't factor 2 or 3 any farther because they are prime numbers. Once you get to the primes in your "tree," they are the "leaves," and you stop factoring in that "branch." So 24 = 2 × 2 × 2 × 3. This is called the **prime factorization** of 24.

```
(root)        24
            /   \
          4   ×   6
         / \     / \
(leaves) 2 × 2 × 2 × 3
```

Examples:

```
  30
 /  \
5 × 6
   / \
  2 × 3
```

The number 5 is prime. It's a "leaf." Once you're done factoring, you "pick the leaves." (You can even circle them to see them better!) So 30 = 2 × 3 × 5.

```
  21
 / \
3 × 7
```

Both 3 and 7 are prime numbers, so we cannot factor them any further.

So 21 = 3 × 7.

```
   66
  /  \
11 × 6
    / \
   2 × 3
```

OR

```
   66
  /  \
2 × 33
    / \
  11 × 3
```

You can start the factoring process any way you want. The end result is always the same: 66 = 2 × 3 × 11.

```
      72
     /  \
    12  × 6
   /  \   / \
3 × 4 × 2 × 3
   / \
  2 × 2
```

Since 72 has a lot of factors, factoring takes several steps.

72 = 2 × 2 × 2 × 3 × 3

We also could have started by writing 72 = 2 × 36 or 72 = 4 × 18.

```
57
/ \
```

How can you get started?
Check:
- Is 57 in any of the times tables?
- Is it divisible by 2? By 3? By 5?

```
65
/ \
```

How can you get started?
Check:
- Is 65 in any of the times tables?
- Is it divisible by 2? By 3? By 5?

1. Factor the following numbers to their prime factors.

a. 18 / \	**b.** 6 / \	**c.** 14 / \
d. 8 / \	**e.** 12 / \	**f.** 20 / \
g. 16 / \	**h.** 24 / \	**i.** 27 / \
j. 25 / \	**k.** 33 / \	**l.** 15 / \

Prime numbers are like the building blocks of all numbers. They are the first and foremost, and other numbers are "built" from them. "Building numbers" is like factoring backwards. We start with the building blocks—the primes—and see what number we get:

```
2 × 5 × 2 × 2
 \ /     \ /
 10  ×   4
  \      /
    40
```

```
2 × 3 × 2 × 3 × 2
 \ /     \ /     |
  6  ×   6   × 2
  |       \    /
  6  ×     12
   \       /
     72
```

```
5 × 2 × 7
 \ /    |
 10  × 7
  \    /
   70
```

```
2 × 7 × 2×3
 \ /    \ /
 14  ×  6
  \     /
   84
```

By using the process above (building numbers starting from primes) you can build ANY whole number there is! Can you believe that?

We can say this in another way: <u>ALL</u> numbers can be factored so the factors are prime numbers. That is sort of amazing! This fact is known as the *fundamental theorem of arithmetic*. And it is indeed fundamental.

So no matter what the number is—992 or 83,283 or 150,282—it can be written as a product of primes.

992 is factored on the right. $992 = 2^5 \times 31$. For 83,283 we get $3 \times 17 \times 23 \times 71$ and $151{,}282 = 2 \times 3^3 \times 11^2 \times 23$.
To find these factorizations, you need to test-divide the numbers by various primes, so it is a bit tedious. Of course, today's computer programs can do the division very quickly.

```
        992
       /   \
    4   ×  248
   / \     /  \
 2 × 2 × 4 × 62
        / \   / \
      2 × 2 × 2 × 31
```

2. Build numbers from these sets of primes.

a. 2 × 5 × 11 \ / |	**b.** 3 × 2 × 2 × 2 \ / \ /	**c.** 2 × 3 × 7 \ / |
d. 11 × 3 × 2 | \ /	**e.** 3 × 3 × 2 × 5 \ / \ /	**f.** 2 × 3 × 17

3. Build more numbers from primes.

a. $2 \times 5 \times 13$	**b.** $13 \times 13 \times 2$	**c.** $19 \times 3 \times 3$

4. Try it on your own! Pick 3 or 4 or more primes (you can even use the same prime several times), and see what number gets built from them.

Puzzle Corner

Ready for a challenge? Use your knowledge of divisibility tests and the calculator to factor these numbers into their prime factors:

a. 2,145 **b.** 3,680 **c.** 10,164

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Chapter 5: Fractions
Introduction

The aim of this chapter is to *review* all fraction arithmetic. The goal is that the student will become "fluent" with basic fraction operations, if he isn't already. The four operations of fractions have been studied in-depth in 5th grade, so the coverage here is quicker than in 5th grade.

For example, the lessons don't always delve into the reasons *why* a certain shortcut works—a lot of that is explained in the fifth grade material. While I consider it very important that the student understands fractions conceptually and understands why certain things are done the way they are done, the material here is building on the material for the earlier grades, where the students have been exposed to those thoughts and ideas.

If the student has a good grasp of fractions already (for example because of having studied the *Math Mammoth Grade 5 Complete Curriculum*), consider assigning only 1/3 - 1/2 of the calculation exercises because he should be able to go through this chapter quickly. However, many students may need the thorough review if they have forgotten these topics since 5th grade, so use your judgment.

The lesson "Comparing Fraction and Decimal Division" is optional. Often, you can solve the same division problem using either fraction or decimal division, if you can convert the numbers from decimals to fractions or vice versa. This lesson just examines the difference between fraction division and decimal division, and best suits advanced and interested students.

The page about fraction equations is also optional and can be omitted.

Besides re-studying fraction operations, students also have several problem-solving lessons to study. A lot of the problems in these lessons review and reinforce already studied concepts, such as ratio and using bar diagrams to solve problems with fractional parts. As a new—and hopefully interesting—application, we study scaling in maps.

The Lessons in Chapter 5

Helpful Resources on the Internet

Fractions and Mixed Numbers

Clara Fraction's Ice Cream Shop
A game in which you convert improper fractions to mixed numbers and scoop the right amount of ice cream flavors on the cone.
www.mrnussbaum.com/icecream/index.html

Simplifying & Equivalent Fractions

Equivalent Fractions
Draw two other, equivalent fractions to the given fraction. Choose either square or circle for the shape.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=80

Fraction Frenzy
Click on pairs of equivalent fractions, as fast as you can. See how many levels you can get!
http://www.learningplanet.com/sam/ff/index.asp

Fresh Baked Fractions
Practice equivalent fractions by clicking on a fraction that is not equal to others.
http://www.funbrain.com/fract/index.html

Fraction Worksheets: Simplifying and Equivalent Fractions
Create custom-made worksheets for fraction simplification and equivalent fractions.
http://www.homeschoolmath.net/worksheets/fraction.php

Equivalent Fractions video
A video by the author that ties in with the equivalent fraction lessons in this book.
http://www.youtube.com/watch?v=NF57T60CSPs

Equivalent Fractions from National Library of Virtual Manipulatives (NLVM)
See the equivalency of two fractions as the applet divides the whole into more pieces.
http://nlvm.usu.edu/en/nav/frames_asid_105_g_2_t_1.html

Addition and Subtraction

MathSplat
Click on the right answer to addition problems (like fractions) or the bug splats on your windshield!
http://fen.com/studentactivities/MathSplat/mathsplat.htm

Adding fractions
Illustrates with pictures finding the common denominator.
http://matti.usu.edu/nlvm/nav/frames_asid_106_g_2_t_1.html

Old Egyptian Fractions
Puzzles to solve: add fractions like a true Old Egyptian Math Cat!
www.mathcats.com/explore/oldegyptianfractions.html

Fraction Bars Blackjack
Computer deals you two fraction cards. You have the option of getting more or "holding". The object is to get as close as possible to 2, without going over, by adding the fractions on your cards.
http://fractionbars.com/Fraction_Bars_Black_Jack/

Action Fraction
A racing game with several levels where you answer questions about adding and subtraction fractions. The levels advance from using like fractions to using unlike fractions and eventually subtraction.
http://funschool.kaboose.com/formula-fusion/number-fun/games/game_action_fraction.html

Fishy Fractions
Select the correct answer and the pelican catches the fish. Options for fraction addition or subtraction, like or unlike denominators, simplifying, comparing, and more.
www.iknowthat.com/com/L3?Area=FractionGame

Comparing Fractions

Comparison Shoot Out
Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal.
www.fuelthebrain.com/Game/play.php?ID=47

Order Fractions
On each round, you drag five given fractions in the correct order.
http://www.bbc.co.uk/schools/ks2bitesize/maths/activities/fractions.shtml

Comparing Fractions - XP Math
Simple timed practice with comparing two fractions.
http://xpmath.com/forums/arcade.php?do=play&gameid=8

Fractional Hi Lo
Computer has selected a fraction. You make guesses and it tells you if your guess was too high or too low.
www.theproblemsite.com/games/hilo.asp

Multiplication and Division

Division of Fractions Conceptually, part 1
A video by the author that ties in with the division lessons in this book. The first part explains about "easy" divisions that can be solved mentally.
http://www.youtube.com/watch?v=41FYaniy5f8

Division of Fractions Conceptually, part 2
A video by the author that ties in with the division lessons in this book. This second part explains about reciprocal numbers and the general "shortcut" for fraction division.
http://www.youtube.com/watch?v=RaT1mDd0_6w

Multiply Fractions Jeopardy
Jeopardy-style game. Choose a question by clicking on the tile that shows the points you will win.
http://www.quia.com/cb/95583.html

Multiply and Reduce Fractions Battleship Game
When you hit the enemy's battleship, you need to solve a fraction multiplication problem.
http://www.quia.com/ba/57713.htm

Fractions Mystery Picture Game
Solve problems where you find a fractional part of a quantity, and uncover a picture.
http://www.dositey.com/2008/math/mistery2.html

Number line bars
Fraction bars that illustrate visually how many times a fraction "fits into" another fraction .
http://nlvm.usu.edu/en/NAV/frames_asid_265_g_2_t_1.html?
open=activities&from=category_g_2_t_1.html

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for fraction addition, subtraction, multiplication, and division.
http://www.homeschoolmath.net/worksheets/fraction.php

Fractions vs. Decimals (and Percents)

Fraction Pie
The user selects the numerator and denominator, and the applet shows the fraction as a pie/rectangle/set model, as a decimal and as a percent.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=45

Comparing Fractions, Decimals, and Percentages
This site has factsheets, a nice matching pairs game, online quiz, and printable worksheets.
http://www.bbc.co.uk/skillswise/numbers/fractiondecimalpercentage/comparing/comparingall3/index.shtm

Fraction Decimal Conversion
Practice fractions vs. decimal numbers online with a matching game, concentration, or flash cards.
www.quia.com/jg/65724.html

Fraction/Decimal Worksheets
Change fractions to decimal numbers or decimal numbers to fractions.
http://www.homeschoolmath.net/worksheets/fraction-decimal.php

Fractions Vs. Decimals Calculator
www.counton.org/explorer/fractions/

Fraction Model
Adjust the the numerator and the denominator, and the applet shows the fraction as a pie/rectangle/set model, as a decimal and as a percent.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=44

All Aspects

Visual Fractions
Great site for studying all aspects of fractions: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by either a number line or a circle with a Java applet. Also couple of games, for example: make cookies for Grampy.
www.visualfractions.com

Conceptua Math
Conceptua Math has free, interactive fraction tools and activities that are very well made. The activities include identifying fractions, adding and subtracting, estimating, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop conceptual understanding of fractions.
www.conceptuamath.com

Who Wants Pizza?
Explains the concept of fraction, addition and multiplication with a pizza example, then has some interactive exercises.
http://math.rice.edu/~lanius/fractions/index.html

Fraction lessons at MathExpression.com
Tutorials, examples, and videos explaining all the basic fraction math topics. Look for the lesson links in the left sidebar.
www.mathexpression.com/understanding-fractions.html

Visual Math Learning
Free tutorials with some interactivity about all the fraction operations. Emphasizes visual models and lets student interact with those.
www.visualmathlearning.com/pre_algebra/chapter_9/chap_9.html

Fractioncity
Make "fraction streets" and help kids with comparing fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. This is not an online activity but has instructions of how to do it at home or at school.
www.teachnet.com/lesson/math/fractioncity.html

Online Fraction Calculator
Add, subtract, multiply or divide fractions and mixed numbers.
www.homeschoolmath.net/worksheets/fraction_calculator.php

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for the four operations with fractions and mixed numbers.
www.homeschoolmath.net/worksheets/fraction.php

Fraction Worksheets: Equivalent Fractions, Simplifying, Convert to Mixed Numbers
Create custom-made worksheets for some other fraction operations.
www.homeschoolmath.net/worksheets/fraction-b.php

Games and Activities to learn Fractions
Commercial games for teaching fractions, including Equivalent Fractions Card Game, Pie and Pizza Fractions, Fractions bingo, teaching fractions with chocolate.
http://lesson-plan.org/lesson-plan/math-lesson-plan.html?offer=frewebhost&pid=0

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Simplifying Before Multiplying

You've already learned to use **factoring** when simplifying.

The example on the right shows simplifying 96/144.

You have also learned how to simplify **"criss-cross."** To simplify 45/150, we cancel the 5s from the numerator and the denominator. Then we simplify 9 and 30 into 3 and 10.

$$\frac{96}{144} = \frac{\cancel{8}^{2} \times \cancel{12}^{1}}{\cancel{12}_{3} \times \cancel{12}_{1}} = \frac{2}{3}$$

$$\frac{45}{150} = \frac{\cancel{5}^{1} \times \cancel{9}^{3}}{\cancel{30}_{10} \times \cancel{5}_{1}} = \frac{3}{10}$$

In a similar manner, you can simplify fractions *before* multiplying. This just saves you one step in writing. Why?

Compare the two examples on the right. If you write the numerator and the denominator multiplications separately, and then simplify, it just adds one extra step of writing. You might as well simplify before writing it out that way since it makes no difference in the result.

$$\frac{7}{\cancel{6}_{2}} \times \frac{\cancel{3}^{1}}{9} = \frac{7}{18}$$

$$\frac{7}{6} \times \frac{3}{9} = \frac{7 \times \cancel{3}^{1}}{\cancel{6}_{2} \times 9} = \frac{7}{18}$$

1. Simplify before multiplying.

E. $\frac{3}{10} \times \frac{1}{3} =$

A. $\frac{5}{6} \times \frac{2}{4} =$

P. $\frac{4}{8} \times \frac{1}{3} =$

O. $\frac{2}{6} \times \frac{5}{7} =$

L. $\frac{2}{9} \times \frac{9}{11} =$

R. $\frac{2}{6} \times \frac{3}{9} =$

M. $\frac{4}{10} \times \frac{1}{3} =$

E. $\frac{3}{10} \times \frac{3}{9} =$

W. $\frac{4}{5} \times \frac{1}{6} =$

I. $7 \times \frac{5}{21} =$

N. $\frac{16}{24} \times 8 =$

S. $\frac{7}{40} \times 15 =$

These problems

$\frac{5}{12}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{21}{8}$	$\frac{5}{3}$	$\frac{2}{15}$	$\frac{1}{6}$	$\frac{2}{11}$	$\frac{1}{10}$	$\frac{16}{3}$	$\frac{5}{21}$	$\frac{2}{15}$

!

You can simplify several times before multiplying.			
$\frac{\cancel{3}^{1}}{15} \times \frac{5}{\cancel{6}_{2}}$	$\frac{\cancel{3}^{1}}{\cancel{15}_{3}} \times \frac{\cancel{5}^{1}}{\cancel{6}_{2}} = \frac{1}{6}$	$\frac{\cancel{3}^{1}}{\cancel{15}_{5}} \times \frac{7}{14}$	$\frac{\cancel{3}^{1}}{\cancel{15}_{5}} \times \frac{\cancel{7}^{1}}{\cancel{14}_{2}} = \frac{1}{10}$
First simplify 3 and 6 into 1 and 2.	Then simplify 5 and 15 into 1 and 3.	First simplify 3 and 15 into 1 and 5.	Then simplify 7 and 14 into 1 and 2.

2. Simplify before you multiply.

a. $\frac{8}{12} \times \frac{6}{12}$ **b.** $\frac{3}{10} \times \frac{2}{18}$ **c.** $\frac{2}{30} \times \frac{10}{11}$

d. $\frac{7}{21} \times \frac{3}{4}$ **e.** $\frac{2}{16} \times \frac{8}{9}$ **f.** $\frac{18}{24} \times \frac{8}{9}$

g. $\frac{5}{36} \times \frac{24}{45}$ **h.** $\frac{16}{30} \times \frac{25}{24}$ **i.** $\frac{14}{25} \times \frac{35}{42}$

3. Try your simplifying skills with multiplying three fractions.

a. $\frac{5}{4} \times \frac{12}{9} \times \frac{3}{15}$ **b.** $\frac{8}{10} \times \frac{15}{27} \times \frac{9}{16}$ **c.** $\frac{1}{18} \times \frac{24}{33} \times \frac{9}{20}$

Puzzle Corner

a. Figure out how this was simplified.

$$\frac{\cancel{15}^{\cancel{5}\,1}}{\cancel{24}_{\cancel{8}\,4}} \times \frac{\cancel{60}^{\cancel{6}\,3}}{\cancel{100}_{\cancel{10}\,2}} = \frac{3}{8}$$

b. Simplify:

$$\frac{60}{48} \times \frac{36}{90} =$$