MATH MAMMOTH

Grade 5-B

Complete Worktext

- Fractions: add and subtract
- Fractions: multiply and divide
- Geometry
- Integers
- Introduction to percent

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www.MathMammoth.com

Contents

Chapter 7: Geometry

Chapter 8: Integers

Chapter 9: Introduction to Percent

Foreword

In part B of the Math Mammoth Grade 5 complete curriculum we study several important topics: fractions, geometry, integers, and a little about percent.

Each chapter has an introduction, which contains notes to the teacher; then follow the actual lessons with problems. Each chapter ends in a review lesson. The chapter tests and cumulative reviews are found in a separate folder and are printed separately. This product also includes an HTML page that you can use to make extra practice worksheets for computation.

In chapter 5, we start the study of fractions, especially focusing on learning to add and subtract unlike fractions. The next chapter continues with fraction topics such as multiplication, simplifying, and the division of fractions. The conversion of fractions to decimals is also included. This essentially completes the main parts of fraction arithmetic, and what is left for 6th grade is mainly review and some deepening of the fraction topics studied here.

Chapter 7 has to do with geometry. The main focus areas are calculating areas of common shapes, including surface areas of some solids, and the volume of a rectangular prism. The chapter also covers classifying quadrilaterals and triangles, similar figures in a coordinate grid, nets of solids, and includes lots of drawing problems.

What follows is a short chapter on integers (chapter 8). We study adding and subtracting integers using several different models as “stepping stones” to the common shortcuts. Then students encounter again the coordinate grid, this time with all four quadrants, and have some fun with moving figures and plotting simple functions.

Lastly, the book includes a short chapter on percent (chapter 9). This is an introduction to the concept of percent with some easy calculations. We will delve into the concept of percent much more in 6th grade.

Concerning Challenging Word Problems

I would heartily recommend supplementing this program with regular practice of challenging word problems and puzzles from other sources. You could do that once a week to once every two weeks - just occasionally as it suits your schedule. The goal of challenging story problems and puzzles is to simply **develop children's logical and abstract thinking and mental discipline**.

I have made lots of word problems for the Math Mammoth curriculum. Those are for the most part multi-step word problems. I have included several lessons that utilize the bar model for solving problems and tried to vary the problems.

Even so, the problems I've created are usually tied to a specific concept or concepts. I feel children can also benefit from problem solving practice where the problems require "out of the box" thinking, or are puzzle-type in nature, or are just different from the ones I have made. Additionally, I feel others are more capable of making very different, very challenging problems.

So I'd like for you to use one or several of the resources below for some different problems and puzzles. Choose something that fits your budget (most of these are free) and that you will like using.

Math Kangaroo Problem Database
Easily made worksheets of challenging math problems based on actual past Math Kangaroo competition problems.
http://www.kangurusa.com/clark/pdb/

Primary Grade Challenge Math by Edward Zaccaro
The book is organized into chapters, with each chapter presenting a type of problem and the ways to think about that problem. Then, there is a series of related story problems to solve, divided into 4 levels. $25, **ISBN** 978-0967991535
You can find this at Amazon.com or various other bookstores.
http://www.amazon.com/dp/0967991536/?tag=homeschoolmath-20

Problem Solving Decks from North Carolina Public Schools
Includes a deck of problem cards for grades 1-8, student sheets, and solutions. Many of these problems are best solved with calculators. All of these problems lend themselves to students telling and writing about their thinking.
http://community.learnnc.org/dpi/math/archives/2005/06/problem_solving.php

Math Stars Problem Solving Newsletter (grades 1-8)
These newsletters are a fantastic, printable resource for problems to solve and their solutions.
http://community.learnnc.org/dpi/math/archives/2005/06/math_stars_news.php

Mathematics Enrichment - nrich.maths.org
Open-ended, investigative math challenges for all levels from the UK. Find the past issues box down in the left sidebar. Choose Stage 2 problems for 5th grade.
http://nrich.maths.org/public/
http://nrich.maths.org/public/themes.php lets you find problems organized by mathematical themes.

Figure This! Math Challenges for Families
Word problems related to real life. They don't always have all of the information but you have to estimate and think. For each problem, there is a hint, other related problems, and interesting trivia. Website supported by the National Council of Teachers of Mathematics.
http://www.figurethis.org/

MathStories.com
Over 12,000 interactive and non-interactive NCTM compliant math word problems, available in both English and Spanish. Helps elementary and middle school children boost their math problem solving and critical-thinking skills. It is a membership site.
http://www.mathstories.com/

"Problem of the Week" (POWs)
Problem of the week contests are excellent for finding challenging problems and for motivation. There exist several:

- **Math Forum: Problem of the Week**
  Five weekly problem projects for various levels of math. Mentoring available.
  http://mathforum.org/pow/
- **Math Contest at Columbus State University**
  Elementary, middle, algebra, and "general" levels.
  http://www.colstate.edu/mathcontest/
- **Aunty Math**
  Math challenges in a form of short stories for K-5 learners posted bi-weekly. Parent/Teacher Tips for the current challenge explains what kind of reasoning the problem requires and how to possibly help children solve it.
  http://www.auntymath.com/
- **Grace Church School's ABACUS International Math Challenge**
  This is open to any child in three different age groups.
  http://www.gcschool.org/pages/program/Abacus.html
- **MathCounts Problem of the Week Archive**
  Browse the archives to find problems to solve. You can find the link to the current problem on the home page.
  http://mathcounts.org/Page.aspx?pid=355
- **Math League's Homeschool Contests**
  Challenge your children with the same interesting math contests used in schools. Contests for grades 4, 5, 6, 7, 8, Algebra Course 1, and High School are available in a non-competitive format for the homeschoolers. The goal is to encourage student interest and confidence in mathematics through solving worthwhile problems and build important critical thinking skills. By subscription only.
  http://www.mathleague.com/homeschool.htm

Chapter 5: Fractions: Add and Subtract
Introduction

In fifth grade students study all aspects of fraction arithmetic. What students have learned in previous years hopefully has built a solid conceptual understanding in their minds, on which we can now build more.

The chapter starts out with lessons on various ways to add and subtract mixed numbers. These are meant partially to review and partially to develop speed in fraction calculations.

The focus of this chapter is on adding and subtracting unlike fractions. Students have already added and subtracted *like* fractions in previous grades. Now it's time to "tackle" the more complex situation.

For that purpose, students need to learn first how to convert fractions into other equivalent fractions. We begin with a visual model of splitting pieces of pie, and from that we develop the common procedure. This skill is then used immediately in the next lessons about adding unlike fractions.

In learning how to add unlike fractions, we begin similarly by using visual models. From the visual and concrete we gradually advance toward the abstract. Several lessons are devoted to understanding the concepts and also to applying this new skill to subtraction as well as to addition and to mixed numbers as well as to fractions.

Next come two lessons on comparing fractions. In these, students learn both mental math methods for comparing fractions and a "brute force" method based on converting to equivalent fractions.

This chapter ends with a lesson on measuring in inches, this year using units as small as 1/16th of an inch.

The Lessons in Chapter 5

Helpful Resources on the Internet

Visual Fractions
A great site for studying all aspects of fractions, including: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by a Java applet with either a number line or a circle. There are also a couple of games, for example: make cookies for Grampy.
http://www.visualfractions.com

Conceptua Math
Conceptua Math has free, interactive fraction tools and activities that are very well made. The activities include identifying fractions, adding and subtracting, estimating, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop conceptual understanding of fractions.
www.conceptuamath.com

Who Wants Pizza?
Explains the concept of fractions, presents addition and multiplication with a pizza example, and then has some interactive exercises.
http://math.rice.edu/~lanius/fractions/index.html

Fraction lessons at MathExpression.com
Tutorials, examples, and videos explaining all the basic fraction math topics. Look for the lesson links in the left sidebar.
http://www.mathexpression.com/understanding-fractions.html

Clara Fraction's Ice Cream Shop
A game in which you convert improper fractions to mixed numbers and scoop the right amount of ice cream flavors into the cone.
http://www.mrnussbaum.com/icecream/index.html

Equivalent Fractions from National Library of Virtual Manipulatives (NLVM)
See how two fractions are equivalent as the applet divides the whole into more pieces.
http://nlvm.usu.edu/en/nav/frames_asid_105_g_2_t_1.html

Equivalent Fractions
Draw two fractions equivalent to the given fraction. Choose either a square or a circle for the basic shape.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=80

Fresh Baked Fractions
Practice equivalent fractions by clicking on a fraction that is *not* equal to the others.
http://www.funbrain.com/fract/index.html

Adding fractions
Uses pictures to illustrate finding the common denominator.
matti.usu.edu/nlvm/nav/frames_asid_106_g_2_t_1.html

Fraction Frenzy
Click on pairs of equivalent fractions as fast as you can. See how many levels you can finish!
http://www.learningplanet.com/sam/ff/index.asp

Fractioncity
Make "fraction streets" and help children with comparing fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. This is not an online activity but has instructions of how to do it at home or at school.
http://www.teachnet.com/lesson/math/fractioncity.html

Visual Math Learning
Free tutorials with some interactivity about all the fraction operations. Emphasizes visual models and lets the student interact with them.
http://www.visualmathlearning.com/pre_algebra/chapter_9/chap_9.html

Comparison Shoot Out
Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal.
http://www.fuelthebrain.com/Game/play.php?ID=47

Old Egyptian Fractions
Puzzles to solve: add fractions like a true Old Egyptian Math Cat!
http://www.mathcats.com/explore/oldegyptianfractions.html

Online Fraction Calculator
Add, subtract, multiply, or divide fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction_calculator.php

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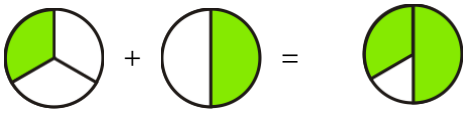
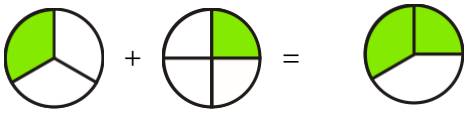
Adding Unlike Fractions 1

Cover the page below the black line. Then try to figure out additions below.

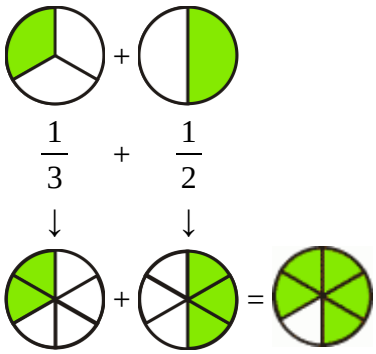


$\frac{1}{3}$ + $\frac{1}{2}$ = What fraction would this be?



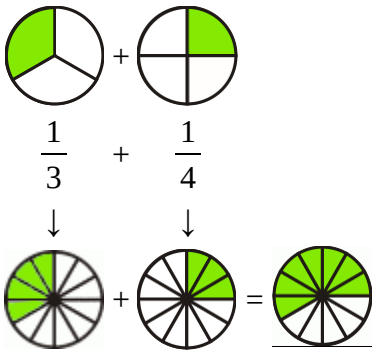
$\frac{1}{3}$ + $\frac{1}{4}$ = What fraction would this be?

Did you solve the problems above? Study the pictures below for solutions. Discuss them with your teacher.



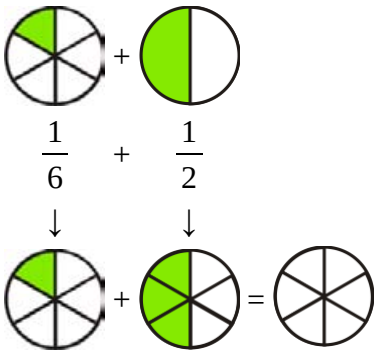
$\frac{1}{3}$ + $\frac{1}{2}$ ↓ ↓

$\frac{2}{6}$ + $\frac{3}{6}$ = $\frac{5}{6}$



$\frac{1}{3}$ + $\frac{1}{4}$ ↓ ↓

$\frac{4}{12}$ + $\frac{3}{12}$ = $\frac{7}{12}$

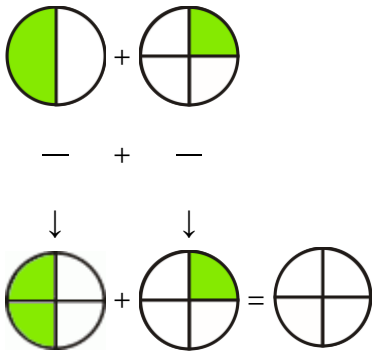


$\frac{1}{6}$ + $\frac{1}{2}$ ↓ ↓

— + — = —

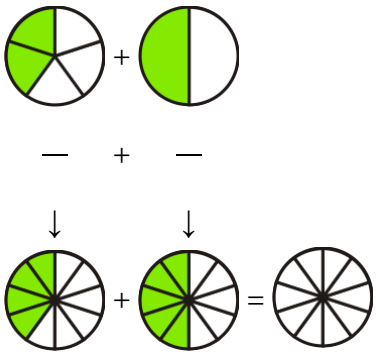
To add unlike fractions, first convert them to _____ fractions. Then add.

1. Write the fractions, convert them into equivalent fractions, and then add them. Color the missing parts.



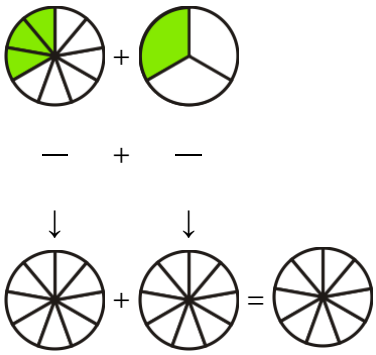
— + — ↓ ↓

a. — + — = —



— + — ↓ ↓

b. — + — = —



— + — ↓ ↓

c. — + — = —

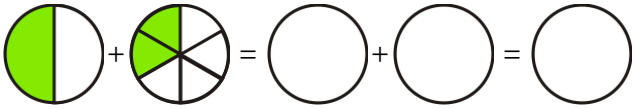
Let's write the addition all on the same line now (horizontally).

$$\frac{3}{10} + \frac{2}{5} = \frac{3}{10} + \frac{4}{10} = \frac{\square}{10}$$

2. Change these to equivalent fractions first and then add them. Each box below has TWO problems. In the bottom problem, you need to figure out what kind of pieces to use, but the *top* problem will help you do that!

a.

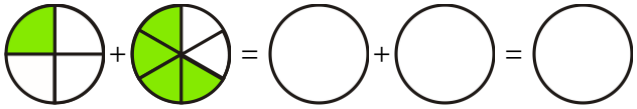
$$\frac{1}{2} + \frac{1}{6} = \frac{\square}{6} + \frac{1}{6} =$$



+ = + =

b.

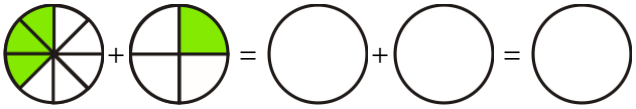
+ = + =



+ = + =

c.

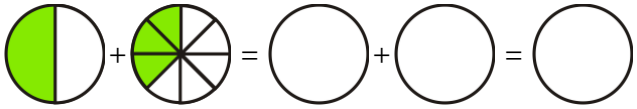
+ = + =



+ = + =

d.

+ = + =



+ = + =

3. Look at the problems above in exercise (2). What kind of parts did the fractions have? What kind of parts did you use in the final addition? Fill in the table.

	Types of parts to add:	Converted to:
a.	2nd parts + 6th parts	6th parts
c.	8th parts + 4th parts	___ parts

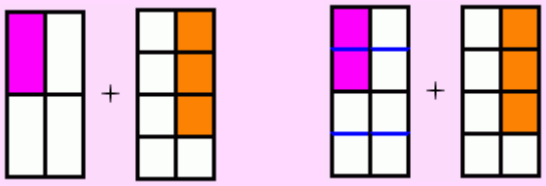
	Types of parts to add:	Converted to:
b.	4th parts + 6th parts	___ parts
d.	2nd parts + 8th parts	___ parts

4. In the problems below, split the parts (as in the example at the right) so that both fractions will have the same kind of parts.

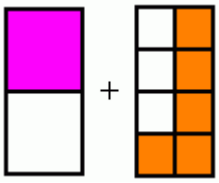
Write an addition sentence.

These pictures change into... ...these.

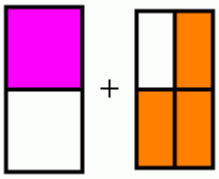
Example:



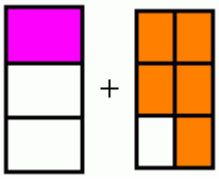
$\frac{1}{4} + \frac{3}{8}$ $\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$



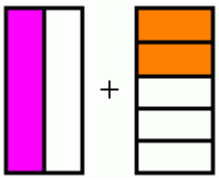
a. — + — =



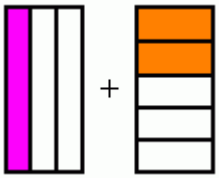
b. — + — =



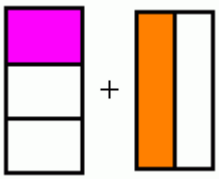
c. — + — =



d. — + — =



e. — + — =



f. — + — =

5. Fill in the table based on the problems above in exercise (4).

	Types of parts to add:	Converted to:
a.	2nd parts + 8th parts	___ parts
b.	2nd parts + 4th parts	___ parts
c.	3rd parts + 6th parts	___ parts

	Types of parts to add:	Converted to:
d.	2nd parts + 5th parts	___ parts
e.	3rd parts + 5th parts	___ parts
f.	3rd parts + 2nd parts	___ parts

6. Now think: How can you know into what kind of parts to convert the fractions that you are adding? Can you see any patterns or rules in the table?

7. **Challenge:** If you think you know what kind of parts to convert these fractions to, then try these problems. Don't worry if you don't know how to do them—we will study this more in the next lesson.

a. $\frac{1}{2} + \frac{2}{3} =$	**b.** $\frac{2}{5} + \frac{2}{3} =$	**c.** $\frac{1}{3} + \frac{1}{4} =$

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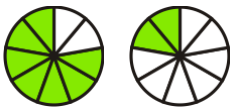
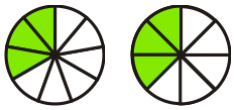
Comparing Fractions 1

Sometimes it is easy to know which fraction is greater.

With like fractions, all you have to do is to check which fraction has more "slices," and that fraction is greater.



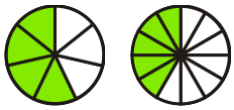
$\frac{7}{9} > \frac{2}{9}$



$\frac{3}{9} < \frac{3}{8}$

If both fractions have the same number of pieces, then the one with bigger pieces is greater.

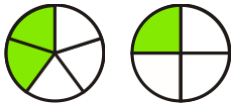
Sometimes you can see that one fraction is less than 1/2 and the other is more than 1/2. Here, 4/7 is clearly more than 1/2, and 5/12 is clearly less than 1/2.



$\frac{4}{7} > \frac{5}{12}$

$\frac{6}{5} > \frac{9}{10}$

Any fraction that is bigger than one must also be bigger than any fraction that is less than one. Here, 6/5 is more than 1, and 9/10 is less than 1.



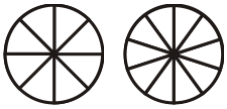
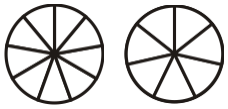
$\frac{2}{5} > \frac{1}{4}$

If you can imagine the pie pictures in your mind, then you can "see" which fraction is bigger. For example, it's easy to see that 2/5 is more than 1/4.

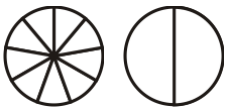
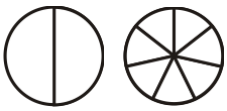
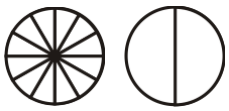
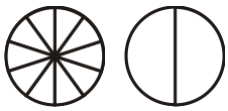
1. These are like fractions. Compare them, and write > or < .

a. $\frac{8}{11}$ $\frac{4}{11}$ **b.** $\frac{21}{16}$ $\frac{25}{16}$ **c.** $\frac{4}{20}$ $\frac{2}{20}$ **d.** $\frac{49}{100}$ $\frac{61}{100}$

2. These fractions have the same number of pieces. Compare them, and write > or <.

 **a.** $\frac{1}{8}$ $\frac{1}{10}$	 **b.** $\frac{3}{9}$ $\frac{3}{7}$	**c.** $\frac{2}{11}$ $\frac{2}{5}$	**e.** $\frac{7}{4}$ $\frac{7}{6}$
		d. $\frac{5}{14}$ $\frac{5}{9}$	**f.** $\frac{1}{20}$ $\frac{1}{8}$

3. Compare these fractions to one half. Then write >, <, or =.

 **a.** $\frac{4}{9}$ $\frac{1}{2}$	 **b.** $\frac{1}{2}$ $\frac{4}{7}$	 **c.** $\frac{7}{12}$ $\frac{1}{2}$	 **d.** $\frac{6}{10}$ $\frac{1}{2}$
e. $\frac{1}{2}$ $\frac{3}{4}$	**f.** $\frac{3}{6}$ $\frac{1}{2}$	**g.** $\frac{1}{2}$ $\frac{5}{8}$	**h.** $\frac{5}{11}$ $\frac{1}{2}$

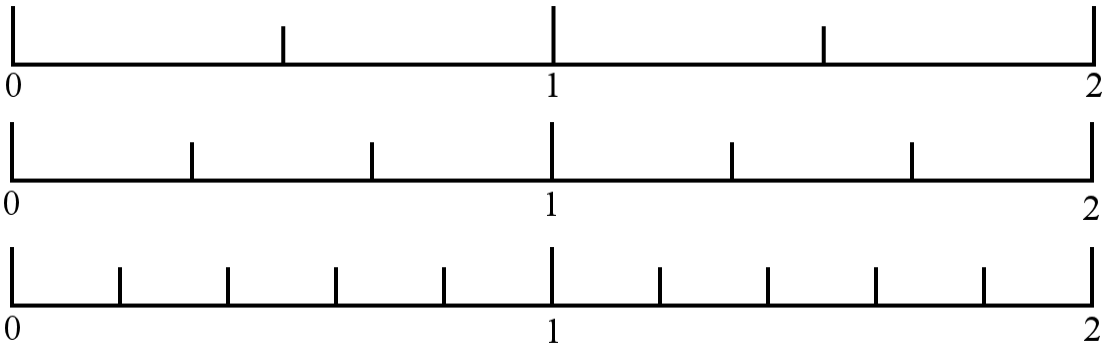
4. Compare each fraction to one. Then write >, <, or = in the box.

a. $\frac{8}{7}$ ☐ $\frac{3}{3}$ **b.** $\frac{4}{4}$ ☐ $\frac{9}{11}$ **c.** $\frac{6}{5}$ ☐ $\frac{3}{4}$ **d.** $\frac{7}{8}$ ☐ $\frac{8}{7}$ **e.** $\frac{12}{9}$ ☐ $\frac{8}{11}$

5. Compare these fractions by imagining the pies in your mind.

a. $\frac{3}{4}$ ☐ $\frac{5}{6}$ **b.** $\frac{1}{3}$ ☐ $\frac{2}{8}$ **c.** $\frac{1}{3}$ ☐ $\frac{3}{9}$ **d.** $\frac{5}{6}$ ☐ $\frac{7}{8}$ **e.** $\frac{5}{8}$ ☐ $\frac{3}{4}$

6. Here are three number lines that are divided respectively into halves, thirds, and fifths. Use them to help you put the given fractions in order, from the least to the greatest.



a. $\frac{1}{3}, \frac{2}{5}, \frac{2}{3}, \frac{1}{5}, \frac{1}{2}$

____ < ____ < ____ < ____ < ____

b. $\frac{7}{5}, \frac{3}{2}, \frac{4}{3}, \frac{6}{5}, \frac{2}{2}$

____ < ____ < ____ < ____ < ____

7. For each pair of fractions, find one that is between them. Any such fraction will do!
(Hint: You can visualize pies in your mind, or convert the fractions into like fractions.)

a. $\frac{1}{6} < \_\_ < \frac{1}{3}$ **b.** $\frac{2}{3} < \_\_ < \frac{7}{8}$ **c.** $\frac{3}{8} < \_\_ < \frac{1}{2}$

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Chapter 6: Fractions: Multiply and Divide
Introduction

This is another long chapter devoted solely to fractions. It rounds out our study of fraction arithmetic. (If you feel that your student would benefit from taking a break from fractions for a while, then you can optionally have him or her study chapter 7 on geometry in between chapters 5 and 6.)

We start out by simplifying fractions. Since this process is the opposite to making equivalent fractions, studied in chapter 5, it should be relatively simple for students to understand. We also use the same visual model, just backwards: This time the pie pieces are joined together instead of split apart.

Next comes multiplying a fraction by a whole number. Since this can be solved by repeated addition, it is not a difficult concept at all. In order to teach the concept, multiplying a fraction by a fraction is first explained as taking a certain part of a fraction. After that, students are shown the usual shortcut for multiplication of fractions.

Simplifying before multiplying is a process that is actually not absolutely necessary for 5th graders, but I have included it here because it prepares students for the same process in future algebra studies and because it makes fraction multiplication easier. Also, I have tried to include explanations of *why* the methods work. Most of these explanations are actually *proofs*. I feel it is a great advantage for students to get used to mathematical reasoning and proof methods well before they start high school geometry.

Then we apply fraction multiplication to calculating the area of rectangles and to multiplying mixed numbers.

Next comes the division of fractions. Fraction division is presented gradually in *five* separate lessons to totally avoid just "announcing" it as one simple shortcut rule. This gradual approach in several steps leads to understanding the concept involved and provides justification for the shortcut.

The first step is dividing a fraction by a whole number, which is easy to understand with the model of dividing pieces of pie among a certain number of people. The next step is "measurement division," in which we consider, "How many times does the divisor fit into the dividend?" For example, $3 \div (1/2)$ can be solved that way. Students will notice a shortcut for divisions of the type $n \div (1/m)$ where n and m are whole numbers. This shortcut is of course leading them towards the ultimate shortcut that works with any kind of fraction division.

The lesson about reciprocal numbers is crucial for understanding why the shortcut for fraction division works. It explains, for example, why 3/4 fits exactly 4/3 times into 1. After this, we generalize the process learned in this lesson to apply to *any* fraction. This also is a common way mathematical proofs are often built: First a specific case is established, and then the general case is derived from it. So the lessons actually illustrate a common technique for mathematical reasoning. It may be slightly disguised, since it is spread over several lessons, but nevertheless students are exposed to the thinking processes used in mathematics.

Some students may not grasp the logical ideas behind the proof. If that happens, make sure they at least get the procedural understanding of fraction division ("To divide by a fraction, multiply by its reciprocal") and are able to perform the calculations. Then come back to these lessons next year and try again.

The last major topic in this chapter is converting fractions into decimals, and it will be studied again in 6th grade. Students need to understand that sometimes we can perform this conversion with mental math and that division (either long division or dividing with a calculator) will *always* do the trick. The skill of converting fractions to decimals is then applied to some problems with measuring units.

Lastly, we present a comparison between ratios and fractions. Ratios is a topic that will be studied a lot in 6th and 7th grades, especially in connection with proportions. We are laying the groundwork for that here.

The Lessons in Chapter 6

Helpful Resources on the Internet

Visual Fractions
A great site for studying all aspects of fractions, including: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by a Java applet with either a number line or a circle. There are also a couple of games; for example: make cookies for Grampy.
http://www.visualfractions.com/

Conceptua Math
Conceptua Math has free, interactive fraction tools and activities that are very well made. The activities include identifying fractions, adding and subtracting, estimating, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop conceptual understanding of fractions.
www.conceptuamath.com

Who Wants Pizza?
These fun tutorials use a pizza example to explain the concept of a fraction and addition and multiplication of fractions. Some interactive exercises are included.
http://math.rice.edu/~lanius/fractions/index.html

Fraction lessons at MathExpression.com
Tutorials, examples, and videos explain all of the basic topics of math with fractions. Look for the lesson links in the left sidebar.
http://www.mathexpression.com/understanding-fractions.html

Fraction Model and **Fraction Pie**
Select the numerator and the denominator, and the applet shows the fraction as a pie/rectangle/set model, as a decimal, and as a percent.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=44
http://illuminations.nctm.org/ActivityDetail.aspx?ID=45

Comparing Fractions, Decimals, and Percentages
This site has fact sheets, a nice matching pairs game, online quiz, and printable worksheets.
http://www.bbc.co.uk/skillswise/numbers/fractiondecimalpercentage/comparing/comparingall3/index.shtm

Fraction Decimal Conversion
Practice fractions and decimal numbers online with a matching game, concentration, and flash cards.
http://www.quia.com/jg/65724.html

Online Fraction Calculator
Add, subtract, multiply or divide fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction_calculator.php

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Multiplying Fractions by Fractions

Most textbooks simply "announce" the rule for multiplying fractions by fractions. This lesson and the exercises in it will let you think and discover WHY the rule works. So please follow all of the explanation and do all of the exercises.

We have studied how to find $\frac{1}{2}$ of a whole number. For example $\frac{1}{2}$ of 24 is $\frac{1}{2} \times 24 = 12$.

- **NOTE:** The word *OF* translates into **MULTIPLICATION**.

Finding $\frac{1}{2}$ *of* any fraction also means multiplying $\frac{1}{2}$ times that fraction!

$\frac{1}{2} \times \frac{1}{3}$ means $\frac{1}{2}$ *of* $\frac{1}{3}$.

Half of the 1/3-piece  is  .

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$\frac{1}{4} \times \frac{1}{3}$ means $\frac{1}{4}$ *of* $\frac{1}{3}$.

A fourth part of the 1/3-piece  is  .

$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

1. The pictures show how much pizza is left, and you must share it equally with one, two, or three other people. Divide the pizza. What kind of part do you get? Write a multiplication sentence.

a. Find $\frac{1}{2}$ of  $\frac{1}{2} \times \frac{1}{4} =$	**b.** Find $\frac{1}{2}$ of  $\times$ =	**c.** Find $\frac{1}{2}$ of  $\times$ =	**d.** Find $\frac{1}{2}$ of  $\times$ =
e. Find $\frac{1}{3}$ of  $\frac{1}{3} \times \frac{1}{2} =$	**f.** Find $\frac{1}{3}$ of  $\times$ =	**g.** Find $\frac{1}{3}$ of  $\times$ =	**h.** Find $\frac{1}{3}$ of  $\times$ =
i. Find $\frac{1}{4}$ of 	**j.** Find $\frac{1}{4}$ of  $\times$ =	**k.** Find $\frac{1}{4}$ of  $\times$ =	**l.** Find $\frac{1}{4}$ of  $\times$ =

Shortcut - multiply fractions of the type 1/n

You might have noticed that in the above exercises, all of our fractions were of the form $\frac{1}{n}$ (where n is a whole number), and that we could have just multiplied the denominators to get the new denominator.

$\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$ or $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

We have now studied how to find 1/2 or 1/3 or 1/5 of some fractions. But how about finding some other kind of fractional part? Let's again compare this to finding fractional parts of whole numbers.

Remember? To find $\frac{3}{4}$ of 16, or in other words $\frac{3}{4} \times 16$, you can first find $\frac{1}{4}$ of 16, which is 4.

Then just take that three times, which is 12. In other words, $\frac{3}{4} \times 16 = 12$.

We can use the same exact idea when finding a fractional part *of* another fraction.

Example. Find $\frac{2}{3}$ of $\frac{1}{4}$. First we find $\frac{1}{3}$ of $\frac{1}{4}$, which is $\frac{1}{12}$.

Then, $\frac{2}{3}$ of $\frac{1}{4}$ is double that much, or $\frac{2}{12}$.

$\frac{2}{3} \times$  = 

Example. Find $\frac{4}{5}$ of $\frac{1}{7}$

First we find $\frac{1}{5}$ of $\frac{1}{7}$, which is $\frac{1}{35}$. Then, $\frac{4}{5}$ of $\frac{1}{7}$ is four times that much, or $\frac{4}{35}$.

Multiplying a fraction by a fraction means taking that fractional part *of* the fraction.
It is just like taking a certain part of what is left over when that leftover is already a fraction.

2. The pictures show how much pizza is left, and you get a certain part of the leftovers. How much will you get? Write a multiplication sentence. Color in an answer picture.

a. $\frac{3}{4} \times$  = 

(First find 1/4 of 1/2, then multiply the result by 3.)

$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ and $\frac{1}{8} \times 3 = \frac{\square}{\square}$

b. $\frac{2}{3} \times$  = 

(First find 1/3 of 1/2, then multiply the result by 2.)

$\frac{1}{3} \times \frac{1}{2} = \frac{\square}{\square}$ and $\frac{\square}{\square} \times 2 = \frac{\square}{\square}$

c. $\frac{3}{4} \times$  = 

(First find 1/4 of 1/3, then multiply the result by 3.)

$\frac{1}{4} \times \frac{1}{3} = \frac{\square}{\square}$ and $\frac{\square}{\square} \times 3 = \frac{\square}{\square}$

d. $\frac{2}{3} \times$  = 

(First find 1/3 of 1/3, then multiply the result by 2.)

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$ and $\frac{\square}{\square} \times 2 =$

e. $\frac{2}{5} \times$  = 

(First find 1/5 of 1/2, then multiply the result by 2.)

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$ and $\frac{\square}{\square} \times 2 =$

f. $\frac{4}{5} \times$  = 

(First find 1/$\square$ of 1/$\square$, then multiply the result by $\square$.)

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$ and $\frac{\square}{\square} \times \square =$

A shortcut for multiplying fractions:

Multiply the numerators to get the numerator for the answer.
Multiply the denominators to get the denominator for the answer.

Let's compare.

The roundabout way	The shortcut
$\frac{5}{6} \times \frac{1}{2} = ?$ (First find 1/6 of 1/2, then multiply the result by 5.) $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$ and $\frac{1}{12} \times 5 = \frac{5}{12}$	$\frac{5}{6} \times \frac{1}{2} = \frac{5 \times 1}{6 \times 2} = \frac{5}{12}$
$\frac{2}{8} \times \frac{3}{5} = ?$ (Find 1/8 of 3/5, then multiply that result by 2. And to find 1/8 of 3/5, first find 1/8 of 1/5, and then multiply that by 3.) $\frac{1}{8} \times \frac{1}{5} = \frac{1}{40}$ Multiplied by 3 that's $\frac{1}{40} \times 3 = \frac{3}{40}$ Then, multiplied by 2 that's $\frac{3}{40} \times 2 = \frac{6}{40} = \frac{3}{20}$ (simplified)	$\frac{2}{8} \times \frac{3}{5} = \frac{2 \times 3}{8 \times 5} = \frac{6}{40} = \frac{3}{20}$
In the "roundabout way," we do each multiplication separately. In the shortcut, we can just do them all at once.	

Study the examples on the right.

Remember always to give your final answer as a mixed number in simplified form.

$\Rightarrow \frac{3}{7} \times \frac{4}{9} = \frac{3 \times 4}{7 \times 9} = \frac{12}{63} = \frac{4}{21}$

$\Rightarrow \frac{12}{5} \times \frac{9}{8} = \frac{12 \times 9}{5 \times 8} = \frac{108}{40} = \frac{27}{10} = 2\frac{7}{10}$

3. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a. $\frac{3}{9} \times \frac{2}{9}$ **b.** $\frac{11}{12} \times \frac{1}{6}$ **c.** $8 \times \frac{3}{13}$

d. $9 \times \frac{2}{3}$ **e.** $\frac{2}{9} \times 8$ **f.** $10 \times \frac{5}{7}$

4. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a. $\frac{3}{4} \times \frac{7}{8}$ **b.** $\frac{7}{10} \times \frac{6}{5}$ **c.** $\frac{9}{20} \times \frac{4}{5}$

d. $\frac{2}{5} \times \frac{1}{3} =$ **e.** $\frac{1}{4} \times \frac{2}{7} =$ **f.** $\frac{5}{4} \times \frac{1}{3} =$

g. $\frac{2}{3} \times \frac{3}{4} =$ **h.** $\frac{2}{9} \times \frac{2}{3} =$ **i.** $\frac{3}{5} \times \frac{1}{10} =$

5. There was 1/4 of the pizza left. Marie ate 2/3 of that.
What part of the *original* pizza did she eat?

What part of the *original* pizza is left now?

6. Mary jogs 7/12 miles each day, five days a week.
Calculate how many miles she jogs during a 52-week year.

7. Sally wants to make 1/3 of the recipe at the right. ⇨
How much does she need of each ingredient?

Brownies

3 cups sweetened carob chips
8 tablespoons extra virgin olive oil
2 eggs
1/2 cup honey
1 teaspoon vanilla
3/4 cup whole wheat flour
3/4 teaspoon baking powder
1 cup walnuts or other nuts

Puzzle Corner Find the missing factors.

a. $\times \frac{6}{7} = \frac{1}{7}$ **b.** $\times \frac{1}{4} = \frac{5}{16}$ **c.** $\times \frac{3}{8} = \frac{1}{16}$ **d.** $\times \frac{2}{5} = \frac{3}{10}$

e. $\frac{1}{5} \times \quad = \frac{1}{20}$ **f.** $\frac{1}{5} \times \quad = 1$ **g.** $\frac{3}{8} \times \quad = 1$ **h.** $\frac{5}{6} \times \quad = \frac{1}{3}$

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Chapter 7: Geometry
Introduction

The problems in this chapter involve lots of drawing. Geometry is a "hands-on" subject, and many children like that. Moreover, drawing is an excellent means of achieving the conceptual understanding that geometry requires.

Exercises marked with the symbol " " are meant to be done in the student's notebook or on blank paper.

This geometry chapter starts out with several lessons that review topics studied in previous grades, such as measuring angles, the vocabulary of basic shapes, and using a compass to draw circles. These review lessons also give those children who haven't previously used *Math Mammoth* an opportunity to "catch up."

After the review lessons we study equilateral and isosceles triangles. Students are now able to classify triangles both in terms of their sides and also in terms of their angles. The lesson has several drawing problems and one easy compass-and-ruler construction of an equilateral triangle.

Then we go on to study the seven different terms used for classifying quadrilaterals. Many textbooks concentrate on learning the vocabulary only, but I have also included several problems that require some thought and even one construction, that of a rhombus. Although many state standards dictate that these words for various quadrilaterals be learned during some earlier grade (3rd or 4th), just learning the words—"rhombus," "trapezoid," "kite," and so on—is meaningless until students can also do something with the figures, such as calculate their areas, find their angles, and reason about their properties. For this reason these terms are introduced here in 5th grade instead.

This chapter focuses on calculating areas, beginning with the area of a right triangle, which is always half of the area of the corresponding rectangle. Once students learn to calculate the area of a parallelogram and realize that the principle applies not just to right triangles and rectangles, but that the area of *any* triangle is always half of the area of the corresponding parallelogram, then they can split any polygon into triangles and thus find its area.

Converting Between Area Units is an introductory lesson. This topic will be studied more in higher grades.

Then we study certain aspects of common solids: the volume of a rectangular prism, surface areas of some solids, and nets of common solids. (Printable pages of some of the nets are included at the very end of the book rather than at the end of the chapter.)

The volumes of round-shaped solids, such as cylinders, cones, and spheres, are not studied until middle school, as students first need to study the concept of *pi* (3.1416...) and its relation to the area of a circle. Another limitation is that students cannot yet find the altitude of the triangle from only the lengths of its sides because that requires the Pythagorean Theorem. Because of these limitations, students cannot yet calculate the surface area or volume of most solids, and these calculations are therefore limited here to certain solids only.

The lesson, *Similar Figures in a Coordinate Grid*, gives students an opportunity to review the coordinate grid and familiarize themselves with similar figures. This lesson is introductory in the sense that similar figures will be studied in much more depth when students understand how to solve proportions.

The Lessons in Chapter 7

Helpful Resources on the Internet

Turtle Pond
Guide a turtle to a pond using commands that include turning him through certain angles and moving him specific distances.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=83

Banana Hunt at *Primary Games*
Help the monkey find bananas and learn to estimate angles.
http://www.primarygames.co.uk/pg2/bhunt/bhunt.html

Ladybug Leaf
Guide the ladybug to turn right or left, 90° or 45°, or to move forward or backward.
http://nlvm.usu.edu/en/nav/frames_asid_287_g_2_t_3.html

Ladybug Mazes
Similar to the Ladybug Leaf, but this time you guide the ladybug through a maze.
http://nlvm.usu.edu/en/nav/frames_asid_141_g_2_t_3.html

Polygon Matching Game
http://www.mathplayground.com/matching_shapes.html

Polygon Vocabulary
Another matching game.
http://www.quia.com/cc/2758.html

Shape Explorer
Find the perimeter and area of odd shapes on a rectangular grid.
http://www.shodor.org/interactivate/activities/perimeter/index.html

Patch Tool
An online activity where the student designs a pattern using geometric shapes.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=27

Interactive Tangram Puzzle
Place the tangram pieces so that they form a given shape.
http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html

Tangram Set
Cut out your tangram set by folding paper.
http://tangrams.ca/inner/foldtan.htm

Geometry - Math Warehouse
Detailed lessons about angles, triangles, quadrilaterals, circles, similar triangles, parallelograms, polygons, and trapezoids.
www.mathwarehouse.com/geometry

National Library of Virtual Manipulatives for Interactive Mathematics: Geometry
A collection of interactive activities: Congruent triangles, fractals, geoboard activities, golden rectangle, ladybug leaf, ladybug mazes, platonic solids, tangrams, tessellations, transformations, and more.
http://nlvm.usu.edu/en/nav/category_g_3_t_3.html

Interactive Geometry
at Mathsnet.net. This is by far the most comprehensive and magnificent interactive geometry resource I've found on the Internet! The link above is to the main page. You can use these quick links to access some of the tutorials (there are lots more):
www.mathsnet.net/geometry/index.html

- **Interactive Shape**
  Get a clear and complete understanding of the patterns and properties of shapes with the help of interactive pictures.
  www.mathsnet.net/shape/index.html
- **Three Dimensions**
  View 3-D interactive pictures of Platonic solids, prisms, and pyramids; practice your ability to visualize 3-dimensional objects from all viewpoints; and more.
  www.mathsnet.net/geometry/solid/index.html

Triangle Explorer
Practice finding the area of triangles. NICE!
http://www.shodor.org/interactivate/activities/triangle/index.html

Interior Angles
Nice lesson and explanation about interior angles of polygons.
http://www.coolmath4kids.com/interior.html

Angles
Practice obtuse/right/acute angles and vertical/corresponding/alternate interior or exterior angle concepts.
http://www.shodor.org/interactivate/activities/angles/index.html

Area Of Parallelogram
This Java applet and its explanation illustrate where the formula for the area of a parallelogram comes from.
http://www.cut-the-not.com/Curriculum/Geometry/AreaOfParallelogram.shtml

Dynamic Rectangle and Parallelogram
Drag the sides of a dynamic parallelogram or a rectangle to explore these concepts.
http://standards.nctm.org/document/eexamples/chap5/5.3/index.htm

Length, Perimeter, Area, and Volume of Similar Figures
Use this interactive figure to explore how the scale factor affects the size and the area of similar figures. The discussion provided helps the teacher somewhat, but a specific lesson plan would be more helpful.
http://standards.nctm.org/document/eexamples/chap6/6.3/index.htm

Dynamic Geometry® Explorations
A bunch of interactive geometry activities, along with instructions and questions to guide your exploration. You need a Java-enabled browser.
http://www.keymath.com/DG/dynamic/index.html

Tim's Triangular Page
Lists all the basic facts about triangles, followed by some problems.
http://sakharov.net/triangle.html

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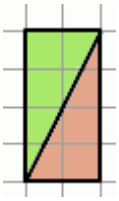
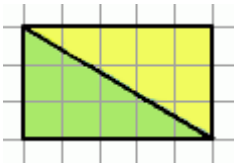
Area of Right Triangles

This rectangle is divided into two right triangles that are ***congruent.*** This means that if you could flip one of them and move it on top of the other, they would match exactly.

The rectangle has an area of 2 × 4 = 8 square units.
Can you figure out what the area of just *one* of the triangles is?

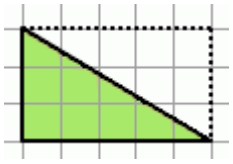


Here the area of the whole rectangle is 3 × 5 = 15 square units.
How could you figure out the area of just one of the triangles?

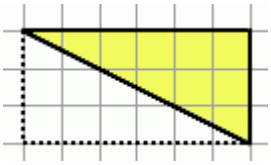


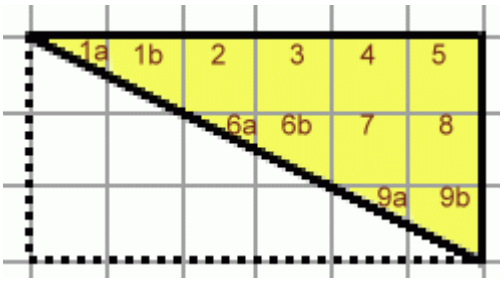
Here the sides of the triangle are 4 and 3 units. The other two sides of the rectangle are drawn with dotted lines.

The area of the *rectangle* is 12 square units. The area of just the triangle is half of that, or 6 square units.



Can you figure out the area of this triangle?





Let's look closer at the last triangle above.
Hopefully you figured out that its area is 9.
To confirm that, we can actually count the little squares in the triangle.

Notice that some of the parts don't cover a complete square, but by combining those we can make whole squares and then count them.

1. Trace the "helping rectangle" for these right triangles. Then find the area of the triangles.

a. _____ square units

b. _____ square units

c. _____ square units

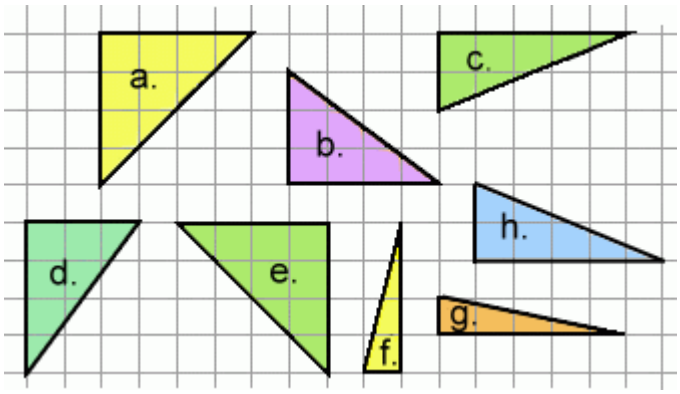
d. _____ square units

e. _____ square units

f. _____ square units

g. _____ square units

h. _____ square units



To find the area of a right triangle, **multiply the lengths of the two sides** that are *perpendicular* to each other (in other words, the two that form the right angle). Then take **half of that**.

This works because the area of a right triangle is exactly _____ of the area of a certain rectangle.

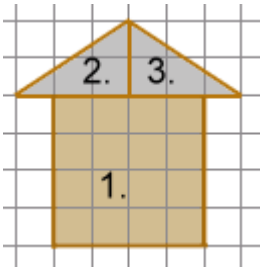
2. Draw a right triangle whose two perpendicular sides are given below, and then find its area.

a. 1 inch and 3 inches

b. 1.2 cm and 5 cm

c. 2 1/2 inches and 1 1/2 inches.

We can find the area of this house-shape in three parts.



1. The square has an area of 4 × 4 = 16 square units.

2. Triangle 2 has perpendicular sides of 3 and 2 units, so its area is (1/2) × 2 × 3 = 3 square units.

3. Triangle 3 is the same shape and size as triangle 2, so its area is also 3 square units.

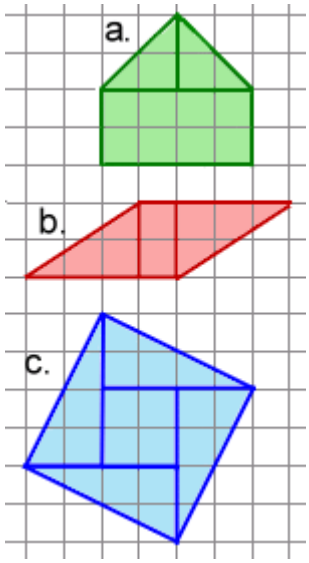
Lastly, add the areas: 16 + 3 + 3 = 22 square units in total.

3. Find the area of these compound shapes. The shapes have already been divided into rectangles and right triangles. Just add the areas of the individual pieces!

a.

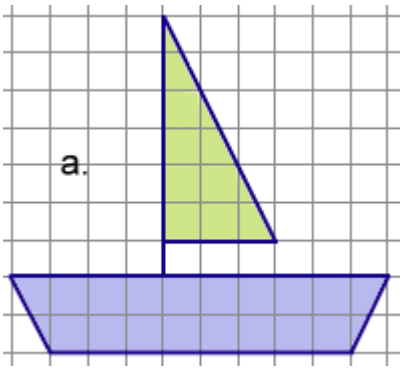
b.

c.

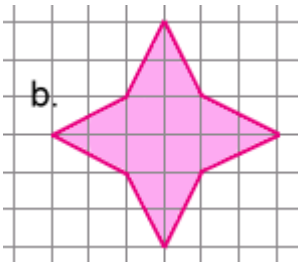


4. Divide the colored shapes into rectangles and right triangles and find their area.

a.



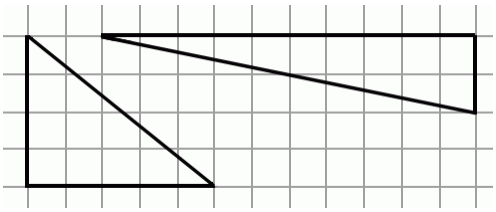
b.



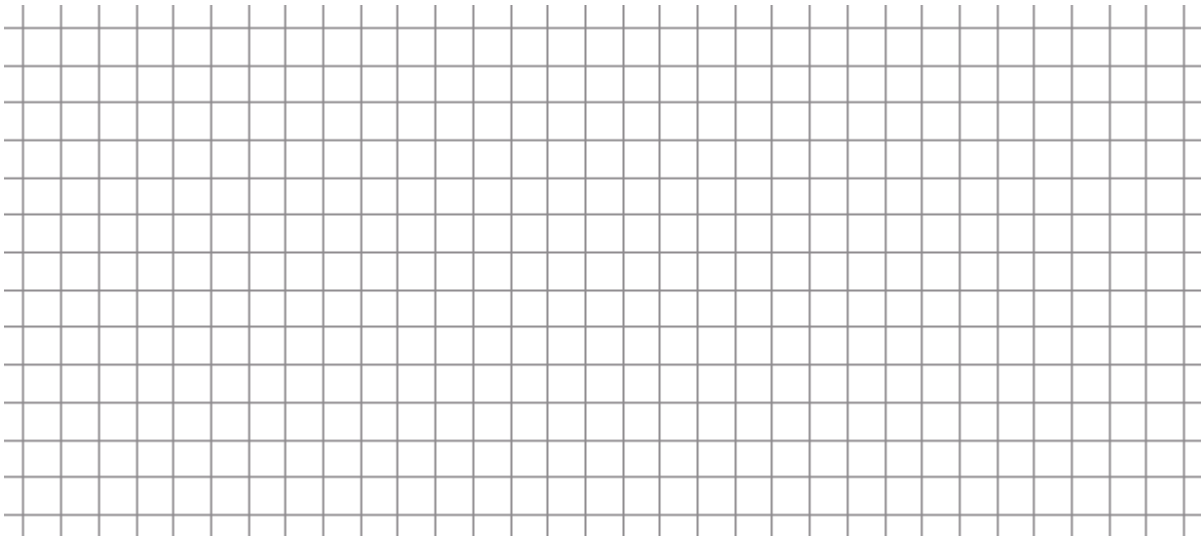
5. **a.** Draw a right triangle whose *area* is 8 square inches.
What can the lengths of the two perpendicular sides be?

b. Draw a right triangle whose *area* is 13 square centimeters.
What can the lengths of the two perpendicular sides be?

6. Here you see two different-shaped right triangles that both have the area of _____ square units.



On the grid below, draw 2 different right triangles that each have an area of:
a. 6 **b.** 9 square units.



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Chapter 8: Integers
Introduction

This short chapter is intended as an introduction to integers. We won't study all the aspects of arithmetic with integers yet—that will happen in 6th and 7th grades.

Integers are introduced using the number line to relate them to the concepts of temperature, elevation, and money. Adding and subtracting integers is presented through several models. The two main models are (1) movements along the number line and (2) positive and negative counters. With the help of these models, students should not only learn the shortcuts, or "rules," for adding and subtracting integers, but also understand *why* these shortcuts work.

This new knowledge is then applied to the coordinate grid. Students learn to locate points in all four quadrants and associate movements in the coordinate grid with addition and subtraction. Lastly, we once again touch on the topic of *functions*. Not only is this last topic preparing students for pre-algebra and algebra courses, but it's also a really nice application of integer addition and subtraction.

This chapter does not cover absolute value or division of integers. Multiplication of integers is only touched on in one of the coordinate grid lessons, and there only the case of a positive integer times a negative integer. The topics left out are studied in 6th grade.

The Lessons in Chapter 8

Helpful Resources on the Internet

Color Chips Addition
The user drags positive or negative chips to the working area, then combines them in pairs to see the sum.
http://matti.usu.edu/nlvm/nav/frames_asid_161_g_2_t_1.html

Number Balls Game
Click on the rotating number balls in ascending order.
http:// themathgames.com/arithmetic-games/integers/order-positive-negative-integers.php

MathCar Racing
Outdrive Funbrain's math car by getting the highest score and practice integer operations.
http://www.funbrain.com/osa/index.html

Choose Operation Game
Choose the range to include negative numbers in this fun game to get practice in how to add, subtract, multiply, and divide integers.
http://www.homeschoolmath.net/operation-game.php

Color Chips Subtraction
Drag individual positive or negative chips or positive-negative chip pairs into working area as instructed, then subtract them.
http://www.matti.usu.edu/nlvm/nav/frames_asid_162_g_2_t_1.html

Integers: Operations with Signed Numbers
Clear and cute presentations of the rules for integer operations, including "The Party in the Mathland" for determining the sign of multiplication and division results. There is a link to practice exercises with instant feedback.
http://amby.com/educate/math/integer.html

Space Coupe to the Rescue
By choosing a positive or negative number, the player controls the vertical position of a spaceship. If the spaceship reaches the same vertical position as a virus pod, then the pod is destroyed.
http://pbskids.org/cyberchase/games/negativenumbers/index.html

Flashcards with Negative Numbers
Interactive flashcards at APlusMath.com.
http://www.aplusmath.com/Flashcards/sub-nflash.html

Adding Real Numbers at Explorelearning.com
An interactive "gizmo" that illustrates the addition of integers on a number line. It has an exploration guide and assessment questions. It's by subscription, but it has a free 30-day trial account.
http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=211

General Coordinates Game
Either type in the coordinates of a displayed point or enter coordinates and the applet will plot the point.
http://www.terragon.com/tkobrien/algebra/topics/orderdpairs/op.html

Graph Mole
A fun game about plotting points in the coordinate plane. Plot the points before the mole eats the vegetables.
http://funbasedlearning.com/algebra/graphing/default.htm

Graphit
A graphing tool that plots both functions and ordered pairs.
http://www.shodor.org/interactivate/activities/graphit/index.html

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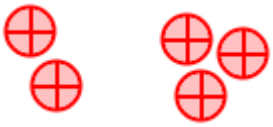
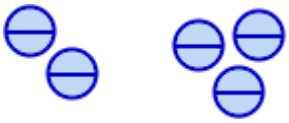
Adding Integers 1

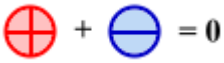
Addition of integers can be modeled using counters.
We'll use red counters with a "+" sign for positives and blue counters with a "–" sign for negatives.



This picture shows the addition, 2 + 3. There is one group of 2 positives and another group of 3 positives. The sum is simply 5.



This picture shows the addition, (−2) + (−3). We *add* negatives and negatives. In total there are five negatives, so the sum is −5.



1 + (−1) = 0

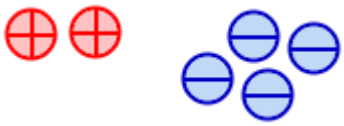
One positive counter and one negative counter *cancel* each other because their sum is zero!

2 + (−2) = 0

Two negatives and two positives also cancel each other.

3 + (−1) = 2

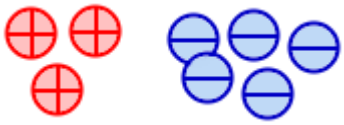
Here, one "positive-negative" pair is canceled, and we are left with 2 positives.



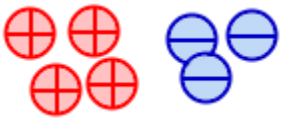
2 + (−4) = −2

Now the negatives outweigh the positives. Pair up two of each, and there are still two negatives left.

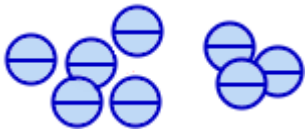
1. Refer to the pictures to solve the equations.



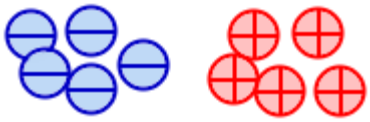
a. 3 + (−5) = _____



b. 4 + (−3) = _____

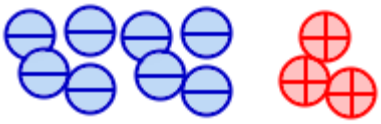


c. (−5) + (−3) = _____



d. (−5) + 5 = _____

e. (−4) + 3 = _____

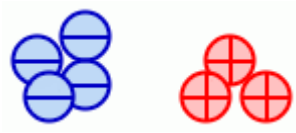
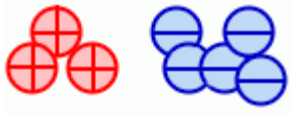
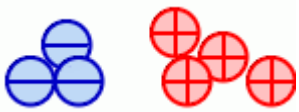
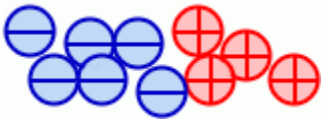
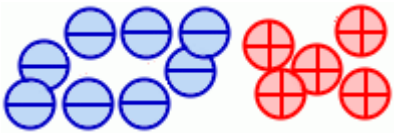


f. (−8) + 3 = _____

2. These equations contain all negative numbers. Solve them.

a. (−5) + (−5) = _____ **b.** (−2) + (−7) = _____ **c.** (−18) + (−5) = _____

3. Write addition sentences (equations) to match the pictures.

 **a.**	 **b.**	 **c.**
 **d.**	 **e.**	 **f.**
 **g.**	 **h.**	 **i.**

4. Rewrite these math sentences in symbols, and solve the resulting addition problems.

a. The sum of five negatives and ten positives.

b. Add −2 and −11

c. Positive 20 and negative 15 added together.

5. Think of the counters. Add.

a. 2 + (−6) = (−2) + 6 =	**b.** (−2) + (−6) = 2 + 6 =	**c.** 4 + (−9) = 9 + (−4) =	**d.** 10 + (−20) = 20 + (−10) =

6. Find the number that is missing from the equations. Think of the counters.

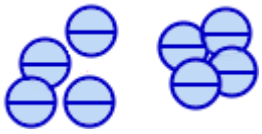
a. 2 + _____ = $^{-}5$ **b.** $^{-}4$ + _____ = 3 **c.** $^{-}5$ + _____ = $^{-}11$

7. You can form four different *addition* expressions (sums) using 3 and 7 (in that order), and the plus and minus symbols.

a. Write those four expressions.

b. Write the expressions in order from the one with greatest value to the one with least value.

A note on notation



We can write an elevated minus sign to indicate a negative number: $^{-}4$.
Or we can write it with a minus sign and parentheses: (−4).
We can even write it without the parentheses if the meaning is clear: −4.

So $^{-}4 + {}^{-}4 = {}^{-}8$ is the same as (−4) + (−4) = (−8), which is the same as −4 + (−4) = −8

You *should* write the parentheses if you have + and −, or two − signs, next to each other.
So don't write "8 + − 4"; write "8 + (−4)." And don't write "3 − −3"; write "3 − (−3)."

8. Think of the counters. Add.

a. 5 + 7 =	**b.** (−5) + 7 =	**c.** $^{-}4 + {}^{-}4$ =	**d.** 12 + $^{-}1$ =
(−5) + (−7) =	5 + (−7) =	$^{-}4 + 4$ =	$^{-}12 + {}^{-}1$ =

9. Solve the problems, and observe the patterns.

a. 5 − 4 =	**b.** $^{-}4 - 0$ =	**c.** $^{-}3 + 0$ =	**d.** $^{-}2 + 2$ =
5 − 5 =	$^{-}4 - 1$ =	$^{-}3 + 1$ =	$^{-}2 + 3$ =
5 − 6 =	$^{-}4 - 2$ =	$^{-}3 + 2$ =	$^{-}2 + 4$ =
5 − 7 =	$^{-}4 - 3$ =	$^{-}3 + 3$ =	$^{-}2 + 5$ =
5 − 8 =	$^{-}4 - 4$ =	$^{-}3 + 4$ =	$^{-}2 + 6$ =

10. Complete the addition sentences, using <u>one positive</u> integer and <u>one negative</u> integer.

a. ___ + ___ = (−1)	**b.** ___ + ___ = 0	**c.** ___ + ___ = 2
___ + ___ = (−1)	___ + ___ = 0	___ + ___ = 2
___ + ___ = (−1)	___ + ___ = 0	___ + ___ = 2

11. Write an *addition or subtraction* sentence to match the temperature change.

a. The temperature was −3°C. Then it rose 4 degrees. Now it is _____. **$^{-}3 + 4 = 1$**

b. The temperature was −5°C. Then it rose 2 degrees. Now it is _____.

c. The temperature was −1°C. Then it rose 7 degrees. Now it is _____.

d. The temperature was 3°C. It dropped 4 degrees. Now it is _____.

e. The temperature was 5°C. It dropped 8 degrees. Now it is _____.

f. The temperature was −1°C. It dropped 5 degrees. Now it is _____.

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Chapter 9: Percent
Introduction

In this chapter we study the basics of percent. The whole chapter is intended as an introduction to the concept of percent. These topics will be studied again, and in more depth, in the sixth grade.

We start out by learning what percent means, by writing percents as decimals and as fractions, and by writing fractions as percents. These conversions tie in very closely with earlier lessons about converting fractions to decimals.

Next we learn how to find a certain percentage of a number. The emphasis is on mental math methods; otherwise the student might start relying on a calculator or on a memorized rule without understanding the underlying concept.

Another important topic is “What Percent?”, or finding the percentage in word problems when other information is given. After some more practice with the basics of percent, students also get to study a comparison of ratios, fractions, and percent.

Although a calculator is certainly handy for calculating percent problems, in this chapter the use of a calculator is allowed only in a few problems. That is because the student needs to understand the concepts first before being allowed to rely on a calculator.

I hope the lessons here will give a good overview of the basics of percent, allowing for an understanding of the concepts. Percent will be an important topic in middle school and in real life, so it will pay to understand it well.

The Lessons in Chapter 9

Helpful Resources on the Internet

Comparing Fractions, Decimals, and Percentages
Fact sheets, a nice pair-matching game, an online quiz, and printable worksheets.
http://www.bbc.co.uk/skillswise/numbers/fractiondecimalpercentage/comparing/comparingall3/index.shtm

Virtual Manipulatives: Percentages
An interactive tool where the student can fill in any two of three "boxes" (for the whole, the part, and the percent), and the app will calculate what should be in the remaining box and show the result visually in two different ways.
http://matti.usu.edu/nlvm/nav/frames_asid_160_g_2_t_1.html

Meaning of Percent; Writing Fractions as Percents; Percents and Proportions
Free percent lessons from Math Goodies. Their CD has many more.
http://www.mathgoodies.com/lessons/vol4/meaning_percent.html
http://www.mathgoodies.com/lessons/vol4/fractions_to_percents.html
http://www.mathgoodies.com/lessons/percent/proportions.html

A Conceptual Model for Solving Percent Problems
An explanation of how to use a 10 × 10 grid to explain the basic concept of percent and also to solve various types of percent problems.
http://illuminations.nctm.org/LessonDetail.aspx?id=L249" target=

Penguin Waiter
A simple game to calculate the correct tip to leave the penguin waiter.
http://www.funbrain.com/penguin/

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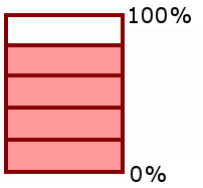
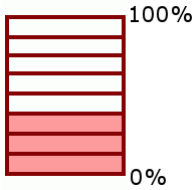
Review

1. Write as percentages, fractions, and decimals.

a. 44% = $\frac{\square}{\square}$ = _____	**b.** ____% = $\frac{7}{100}$ = _____	**c.** ___% = $\frac{\square}{\square}$ = 0.21

2. Write the fractions as percents. Use long division. Round the answers to the nearest tenth of a percent.

a.  Shaded: $\frac{\square}{\square}$ = _____% Unshaded: $\frac{\square}{\square}$ = _____%	**b.**  Shaded: $\frac{\square}{\square}$ = _____% Unshaded: $\frac{\square}{\square}$ = _____%

3. Fill in the table. Use mental math.

Percentage / Number →	**1,300**	**700**	**80**	**48**	**2.4**
1% of the number					
3% of the number					
10% of the number					
25% of the number					

4. **a.** If 11/20 of a certain parcel of land is wasteland, then what percentage of that land is wasteland?

   b. The area of the parcel is 4,500 m^2. Calculate how many square meters of the land is wasteland.

5. What is the discount price if a $16 wall calendar is discounted by 20% ?

6. John weighs 27 kg and Matthew weighs 45 kg. What percentage of Matthew's weight is John's weight?

7. A chain store bought a shipment of 12,000 kg of red, yellow, and green apples. The apples were in a ratio of 2 : 1 : 2 (red to yellow to green).

   a. How many kilograms of apples are green?

   b. How many percent (by weight) of the apples are green?